ICS 161 — Algorithms — Spring 2005 — Final Exam

Please answer the following eight questions on the answer sheets provided. Answers written on other pages or on the wrong sheet will not be scored. Be sure to write your name and student ID on all three answer sheets. You may continue your answers on the back of the same answer sheet. No books, notes, or calculators may be used during the exam.

1. (10 points) Use the Master Method to solve the following two recurrences. Write your answers using O notation.
   (a) $Q(f) = 2Q(f/2) + \log^2(f)$
   (b) $H(m) = 3H(m/2) + m^3$

2. (15 points) The following recursive algorithm sorts a sequence of n numbers. Write down a recurrence describing the running time of the algorithm as a function of n. You do not need to solve your recurrence.

```
def triplesort(seq):
        if n <= 1: return
        if n == 2:
                replace seq by [min(seq),max(seq)]
                return
        triplesort(first 2n/3 positions in seq)
        triplesort(last 2n/3 positions in seq)
        triplesort(first 2n/3 positions in seq)
```

3. (15 points) Suppose you have an undirected graph with weighted edges, and perform a depth-first search, such that the edges going out of each vertex are always explored in order by weight, smallest first. Is the depth first search tree resulting from this process guaranteed to be a minimum spanning tree? Explain why, if it is, or, if it isn't, provide a counterexample (specifying the start vertex for the DFS and showing both trees).

4. (15 points) We are given as input a sequence L of n numbers, and must partition it into as few contiguous subsequences as possible such that the numbers within each subsequence add to at most 100. For instance, for the input [80,-40,30,60,20,30] the optimal partition has two subsequences, [80] and [-40,30,60,20,30]. Let C[i] denote the number of subsequences in the optimal solution of the subproblem consisting of the first i numbers, and suppose that the last subsequence in the optimal solution for all of L has k terms; for instance, for the same example as above, $C[3] = 1$ (the subproblem [80,-40,30] needs only one subsequence) and $k = 5$ (the optimal solution for L has five numbers in the last subsequence). Write a formula showing how to compute C[n] from k and from earlier values of C.

5. (10 points) How much time would the longest common subsequence algorithm described in class use to find the longest common subsequence of two strings, one of which has length s and the other of which has length t? Use O notation.

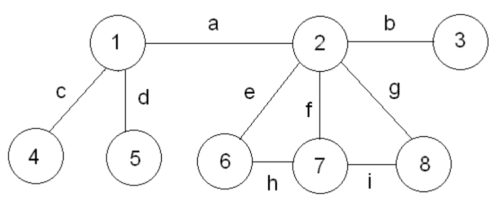
6. (15 points) Suppose you wish to compute the convex hull of n points, all of which have integer coordinates in the range from 1 to n. Describe how to modify the convex hull algorithm described in class so that it solves this problem in $O(n)$ time.

7. (20 points) In the VERTEX COVER decision problem, we are given as input a graph G and a number k, and must test whether G has a vertex cover with k or fewer vertices; that is, a set of k or fewer vertices that are adjacent to all edges in the graph. This problem is NP-complete.

(a) If VERTEX COVER has no polynomial time solution (that is, if P$\neq$NP), can there exist a polynomial time algorithm that takes as input a graph G and finds the vertex cover of G with the smallest number of vertices? Why or why not?

(b) If VERTEX COVER has no polynomial time solution (that is, if P$\neq$NP), can there exist a polynomial time algorithm that takes as input a graph G and finds the vertex cover of G with the largest number of vertices? Why or why not?

8. (20 points) Consider the graph below.



(a) Write the vertices in an optimal vertex cover for this graph.

(b) Write the vertices in the vertex cover produced by the 2-approximation algorithm described in class for this problem, assuming that it considers the edges in the order of the alphabetical ordering of their labels.

(c) What approximation ratio does the algorithm achieve on this particular input?

(d) Why doesn't your answer to (c) contradict the ratio of 2 already proven for this algorithm?

ICS 161 S05 — Answer Sheet 1

Name:

Student ID:

Please answer question 1 in the space below.

Please answer question 2 in the space below.

Please answer question 3 in the space below.

1: 2: 3: 4: 5: 6: 7: 8:
total:

ICS 161 S05 — Answer Sheet 2

Name:

Student ID:

Please answer question 4 in the space below.

Please answer question 5 in the space below.

Please answer question 6 in the space below.

ICS 161 S05 — Answer Sheet 3

Name:

Student ID:

Please answer question 7 in the space below.

Please answer question 8 in the space below.