Delta-Confluent Drawings

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CA-55 @ CA-91, from Terraserver

Confluent drawing

[Dickerson, Eppstein, Goodrich, & Meng, GD'03]

Style of drawing non-planar graphs in a planar way

Vertices are drawn as disks, rectangles, etc as usual

Connections between vertices are drawn as "train tracks", collections of smooth curves meeting tangently at junctions

An edge exists between two vertices iff they are connected by a smooth path along some train track

Previously used e.g. for airline flight maps

Example confluent drawing

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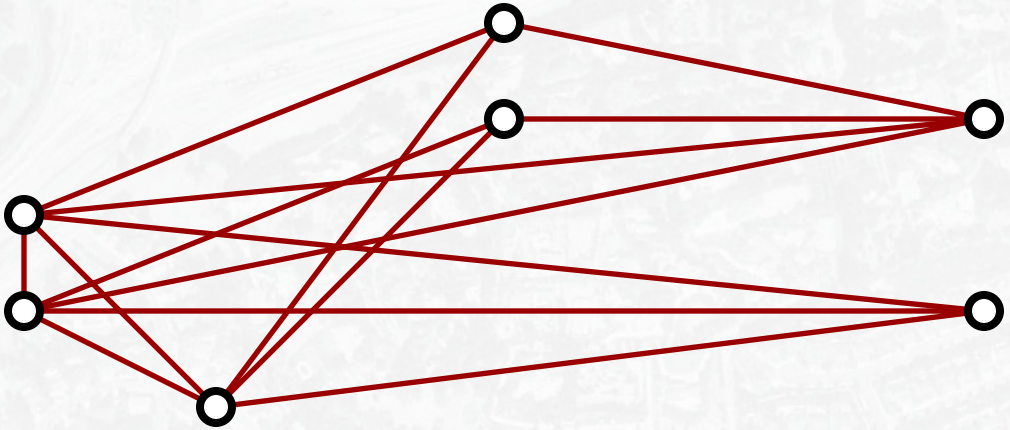


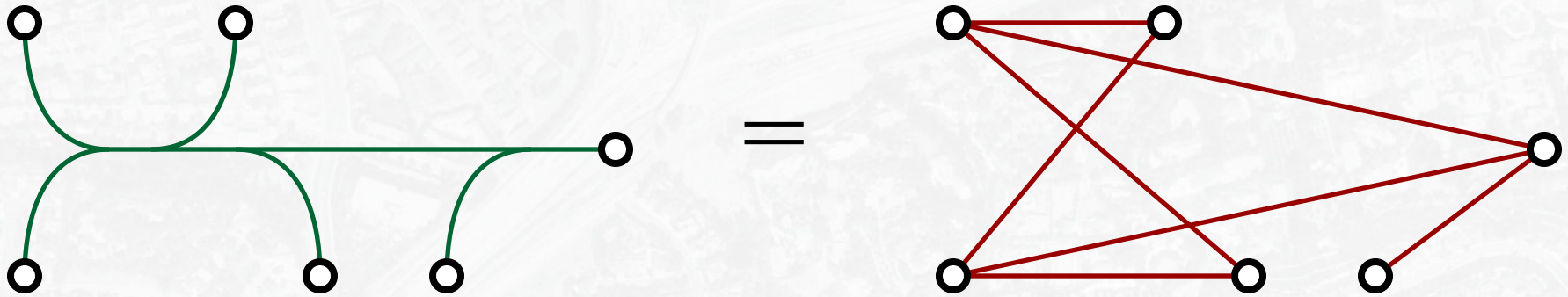
Tree-confluent drawing

[Hui, Schaefer, & Stefankovic, GD'04]

Confluent drawing in which there is a single train track that is topologically a tree (does not disconnect the plane)

Recognizable in O(m+n) time

Tree-confluent graphs are a subclass of chordal bipartite graphs



Research goals in confluent drawing

Determine computational complexity of finding confluent drawings

- Hui et al: at most in NP

Find important graph classes which have confluent drawings

- Dickerson et al:
  - cographs
  - interval graphs
  - complements of trees and cycles
- Hui et al:
  - tree-confluent graphs

Develop heuristics for finding confluent drawings

Combine confluence with aesthetic quality criteria

New results

Generalize tree-confluent drawings by allowing "delta junctions"

Can still be recognized in O(m+n) time

Characterization:

delta-confluent = distance-hereditary

tree-confluent = bipartite distance-hereditary

Allows standard tree-drawing techniques to be applied
in delta-confluent drawing of distance-hereditary graphs
leading to aesthetic, readable confluent drawings

Delta junction

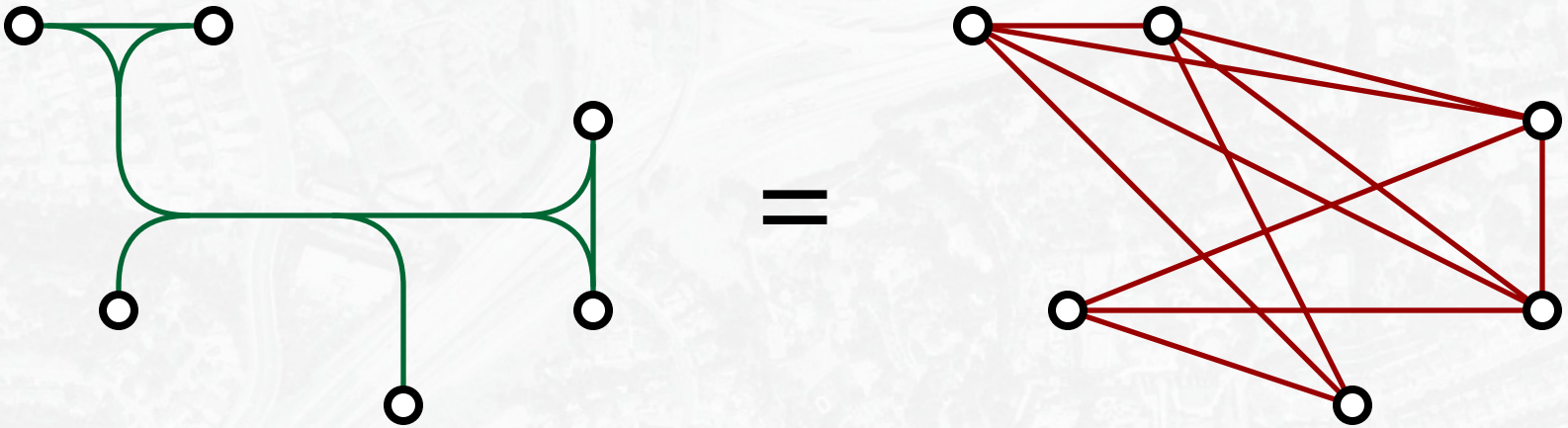
Three components of train track, joined by three confluent junctions

Any path entering the junction must exit on either of the other two tracks

Delta-confluent drawing

Confluent drawing in which there is a single train track
that is topologically a tree except for delta-junctions

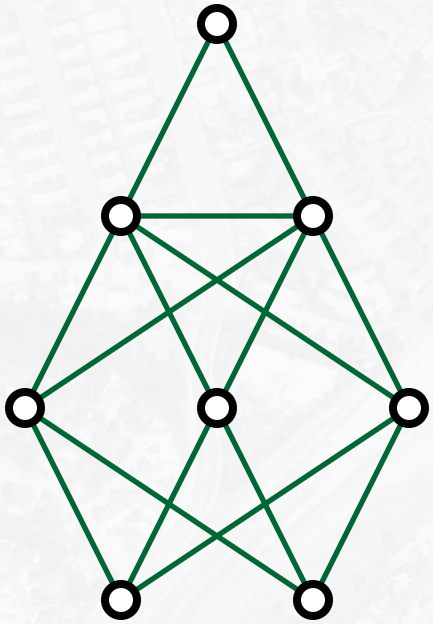
That is, track cuts the plane into one unbounded connected component
plus finitely many small triangles inside delta-junctions



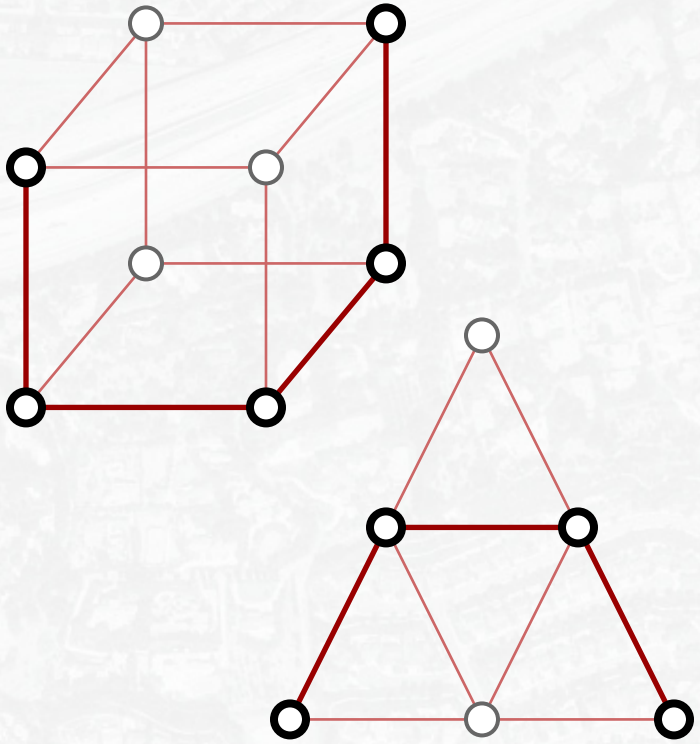
Distance-hereditary graphs

Every shortcut-free path is a shortest path

Equivalently, induced subgraphs have same distances as whole graph



Distance-hereditary



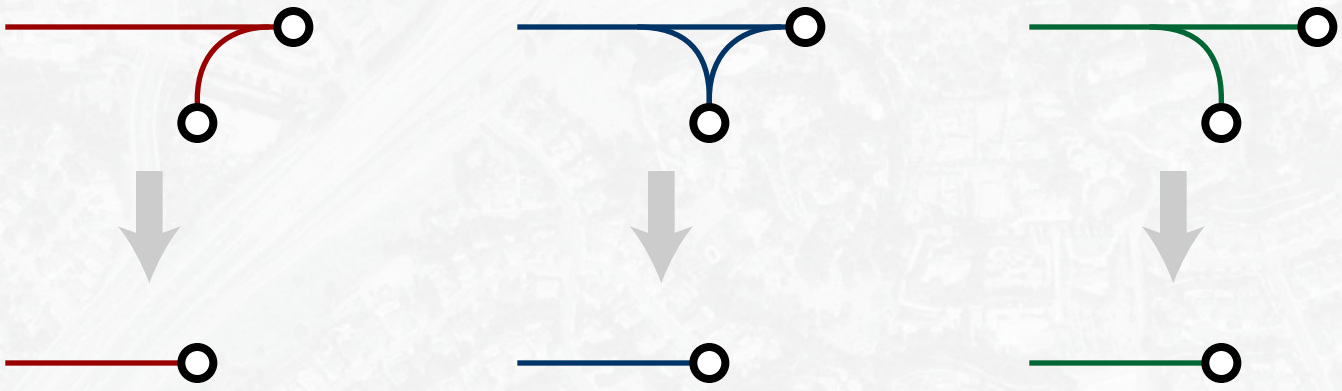
Not distance-hereditary

Delta-confluent = distance-hereditary

Known characterization of distance-hereditary graphs:

They can be reduced to a single vertex by repeatedly
- removing any degree one vertex, or
- merging two vertices with the same sets of neighbors (twins)

Same is true for delta-confluent drawings:
- find two leaves adjacent to same junction, replace by one vertex



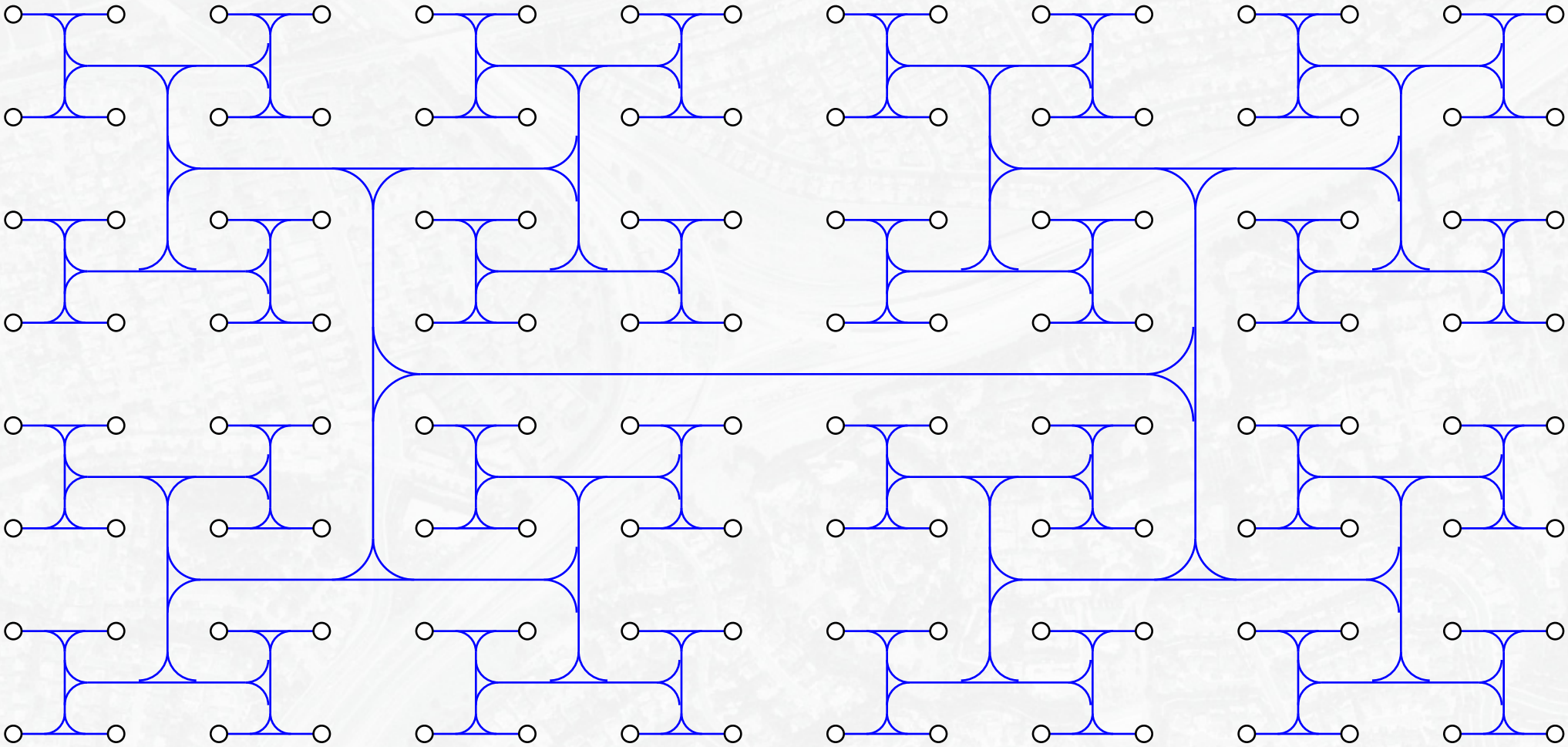
Delta-confluent drawing algorithms

To find a drawing of a distance-hereditary graph:

- Use known distance-hereditary graph recognition algorithms to find sequence of removals and merges reducing to one vertex
- Reverse the sequence, forming an abstract tree with confluent junctions and delta junctions as internal nodes
- Apply any tree layout algorithm to place the tree's nodes
- Replace internal nodes by confluent junctions and delta-junctions

Automatically extends known results on aesthetic criteria for trees (bends, area, etc) to delta-confluent drawings

Example: H-tree layout of complete graph on 128 vertices



Conclusions

Efficient, aesthetic drawing algorithms for an important class of graphs

Exact characterization of graphs drawable delta-confluently

Possible future work

Drawings with more than one confluent tree

Tree-structured confluent drawings with more general junctions

Notions of confluence for directed graphs