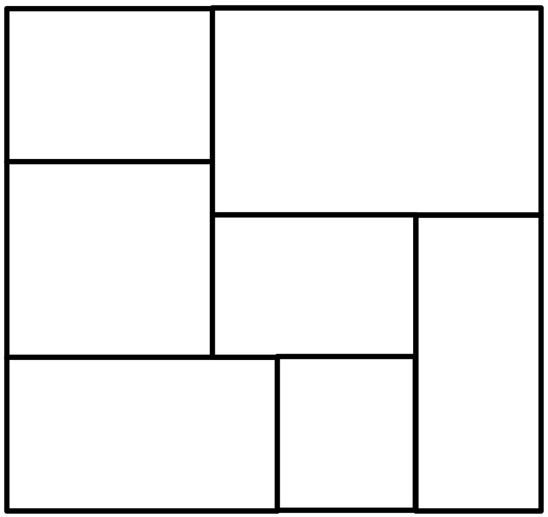
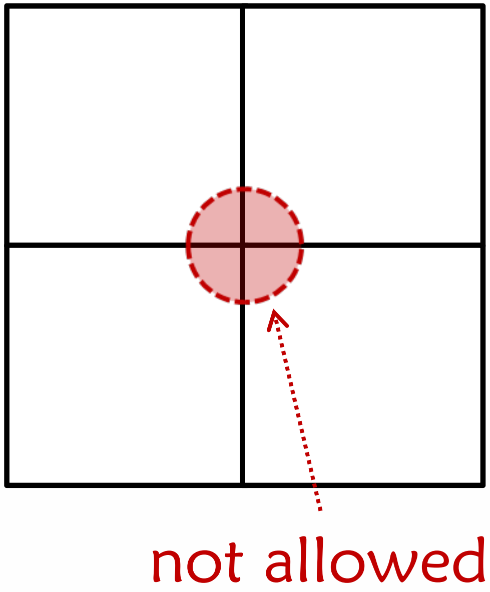
Area-Universal Rectangular Layouts

David Eppstein
University of California, Irvine

Elena Mumford, Bettina Speckmann, and Kevin Verbeek
TU Eindhoven

Rectangular layout

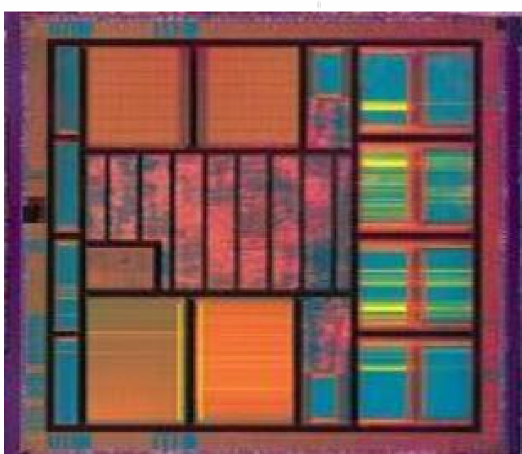




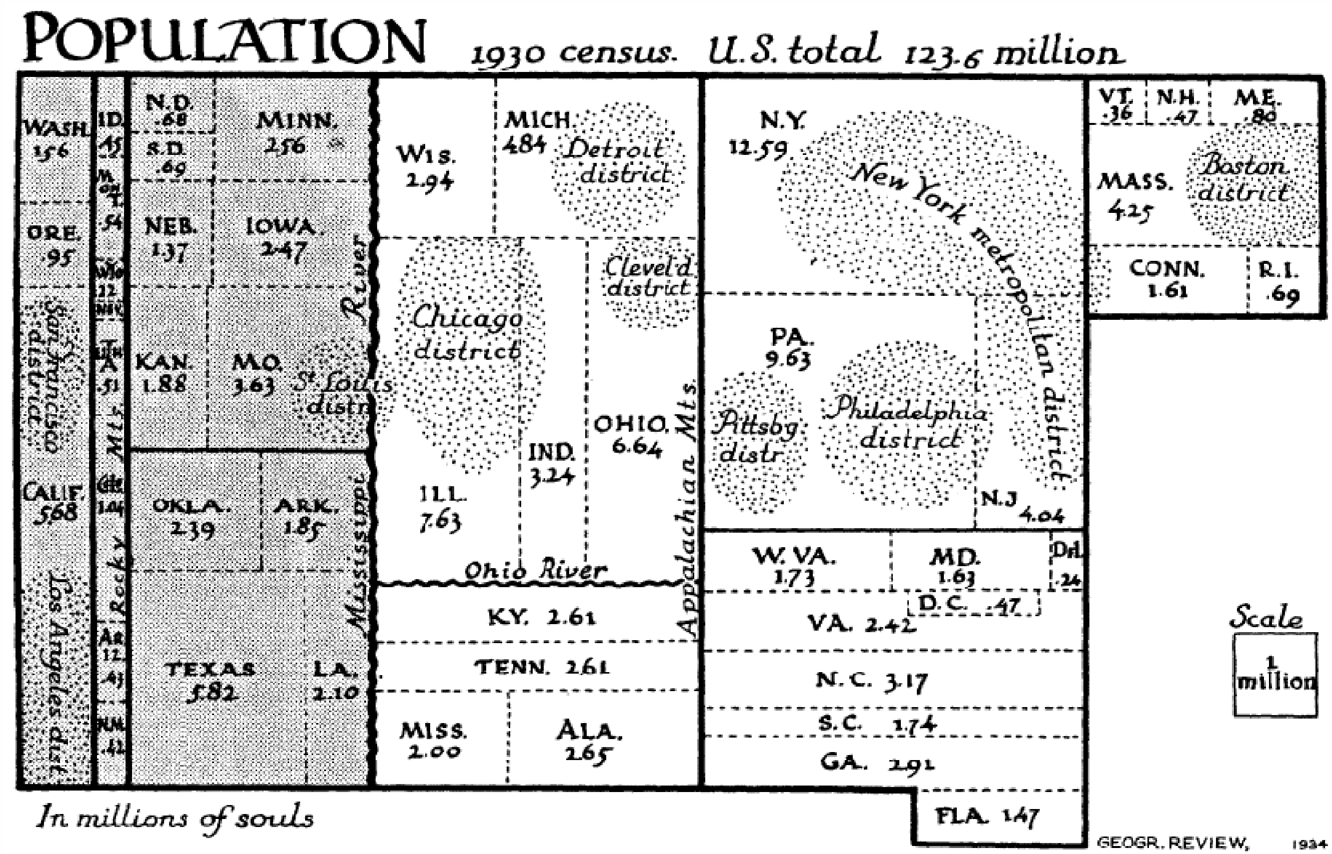
Rectangular layout

partition of a rectangle into finitely many interior-disjoint rectangles, such that no four rectangles meet in one point.

Applications: floor planning



Applications: rectangular cartograms

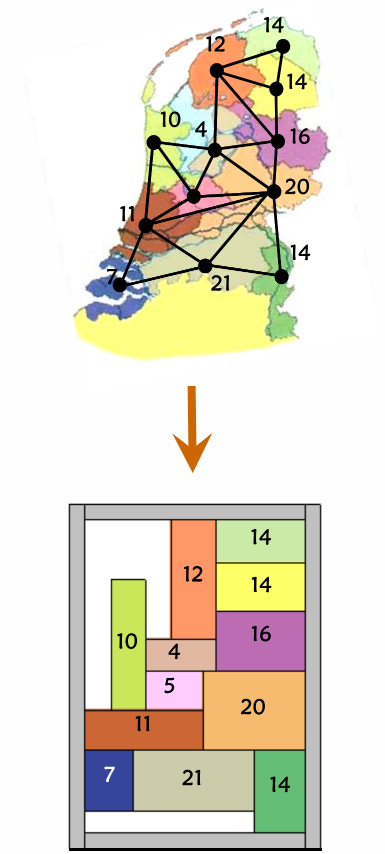


Rectangular Cartograms [Raisz 1934]

visualize statistical data about sets of regions;
regions are rectangles;
area proportional to some geographic variable

Rectangular cartograms

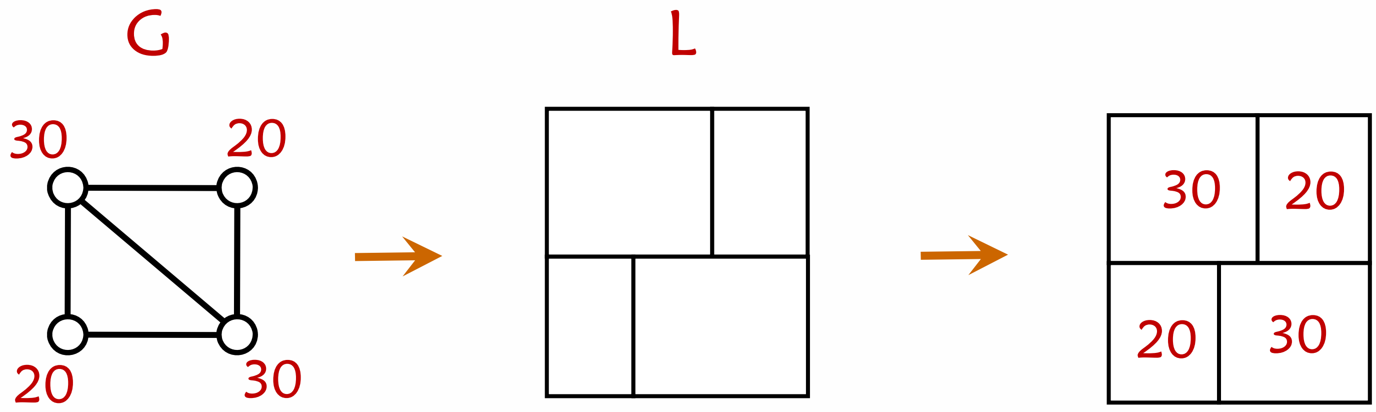
Given a plane triangulated graph G = (V,E) and a positive weight for each vertex.



Construct a partition of a rectangle into rectangular regions

- G is the dual graph of the partition (that is, the partition is a rectangular dual of G)
- The area of each region = the weight of the corresponding vertex

Constructing a cartogram



1. Find a rectangular dual L for G
2. Give rectangles correct areas

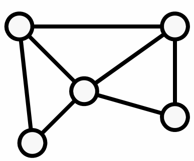
Rectangular dual

[Kozminski & Kinnen '85]

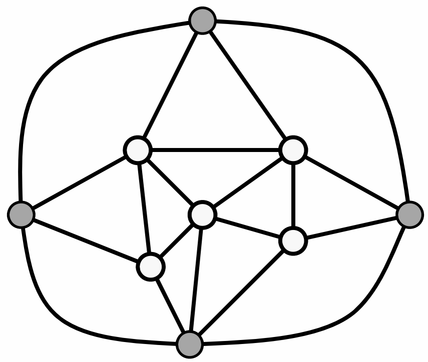
A planar graph G has a rectangular dual $\Leftrightarrow$ we can complete with four outer vertices to obtain a graph E(G) s.t.

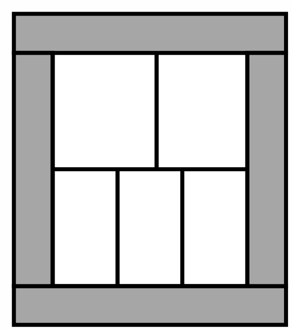
1. every interior face of E(G) is a triangle
2. the exterior face of E(G) is a quadrangle
3. E(G) has no separating triangles

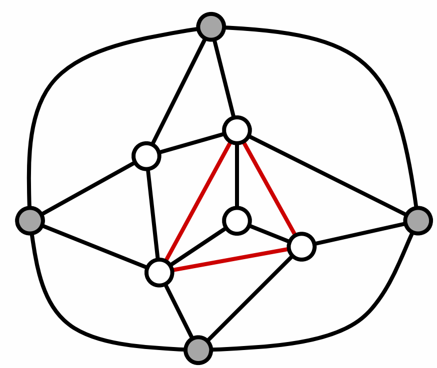
G



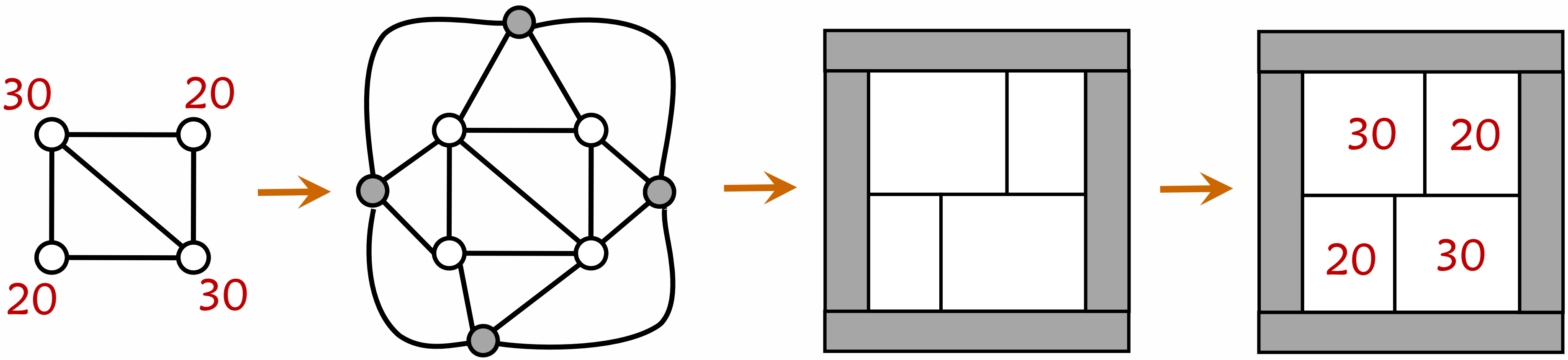
E(G)







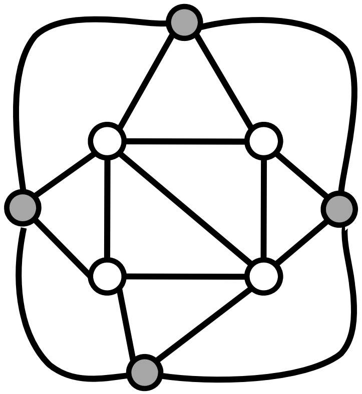
Constructing a cartogram



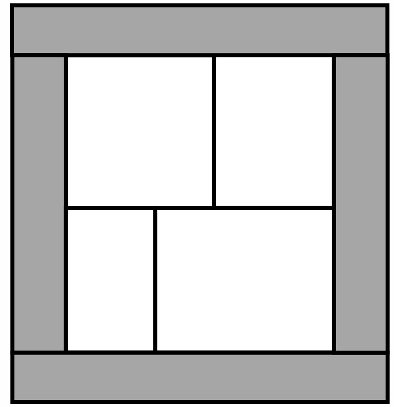
1. Find a rectangular dual L for G
2. Give rectangles correct areas = turn it into an equivalent layout whose regions have given areas

Equivalent layouts

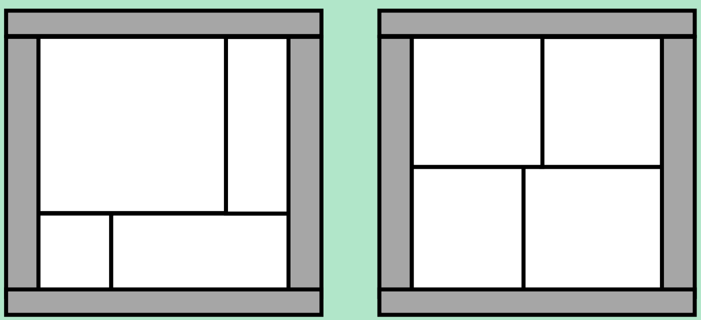
E(G)



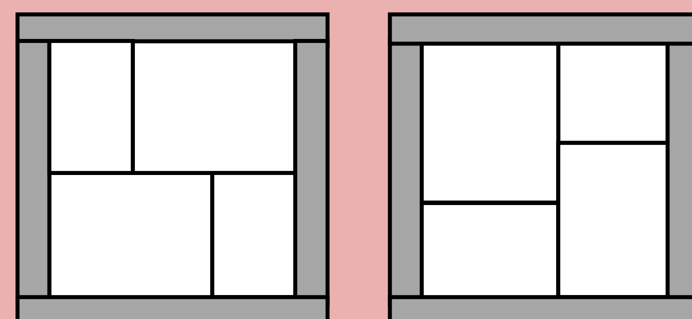
L



equivalent to L



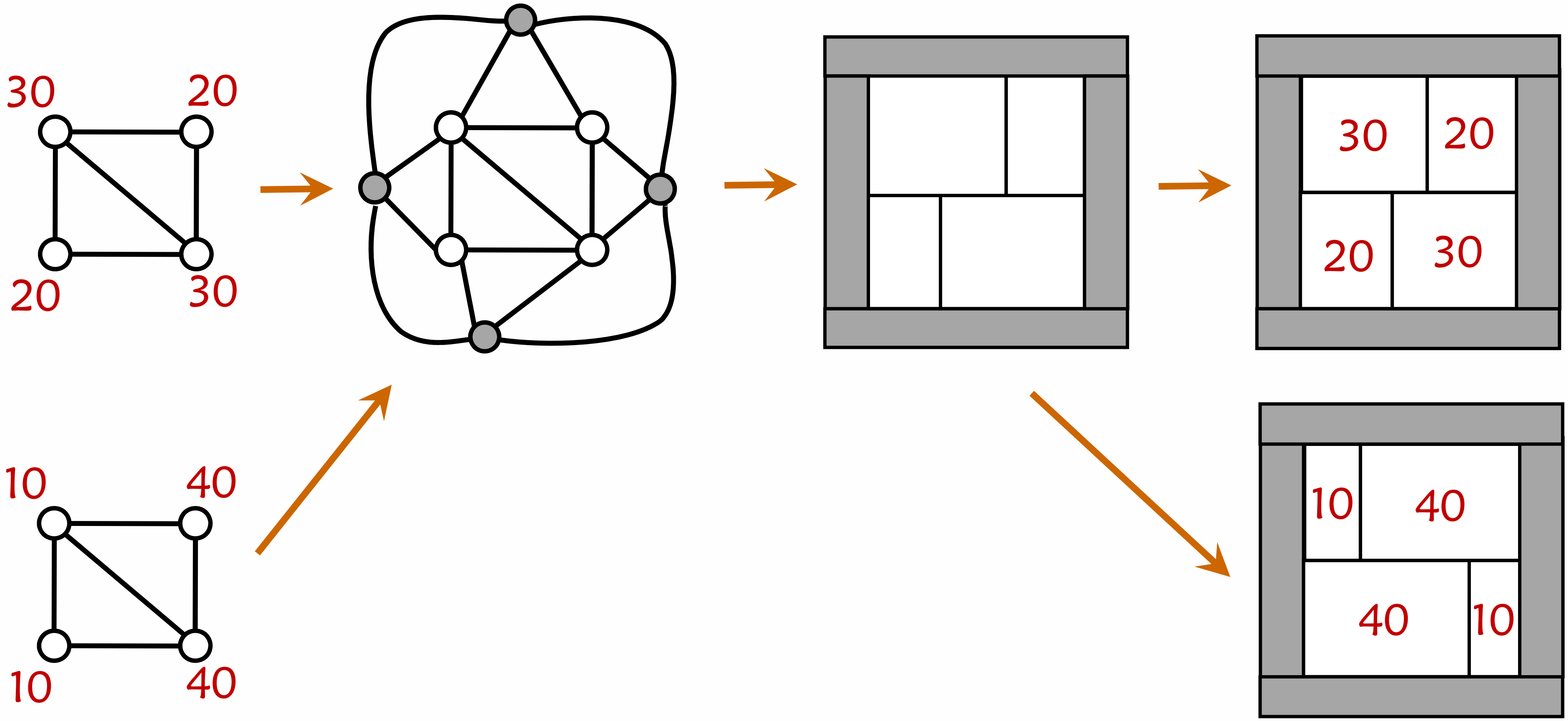
NOT equivalent to L



Equivalent layout

a rectangular dual of L such that the adjacencies of the regions have the same orientations

Constructing a cartogram

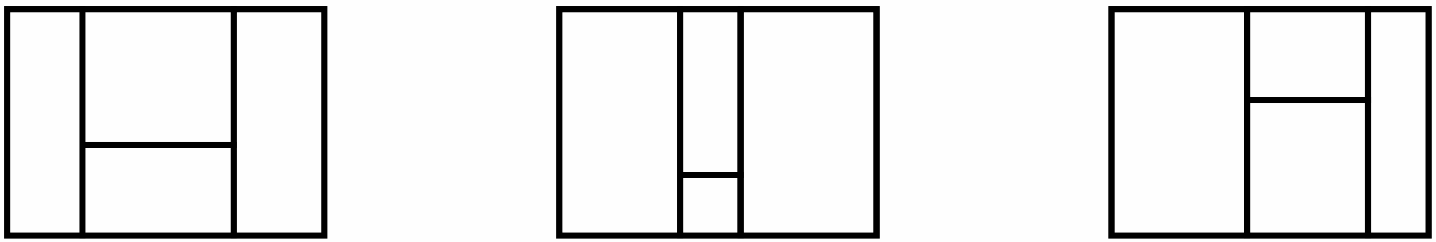


Solution does not always exist

When it does it is unique [Wimer, Koren, and Cederbaum '88]

Finding a suitable layout

- There are potentially exponentially many rectangular duals for a given graph
- There are layouts that "work" for any set of weights:



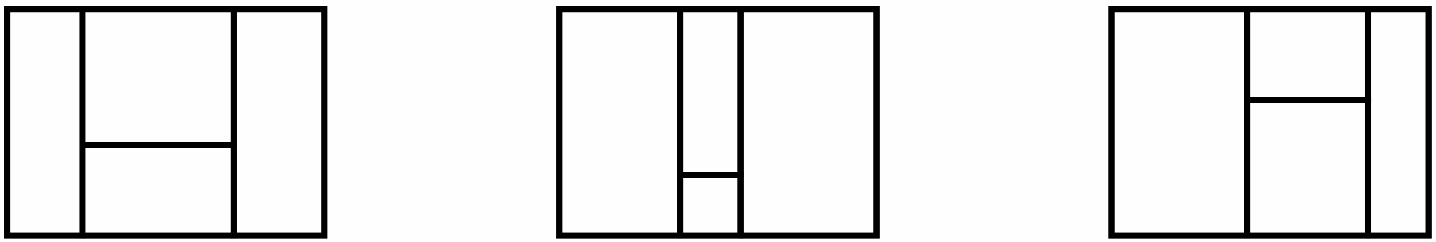
Area-universal layout L

for every choice of weights for the regions of L there is a layout L' equivalent to L such that the areas of rectangles in L' are equal to the given weights.

Finding a suitable layout

Theorem

A layout is area-universal, if an only if it is one-sided.



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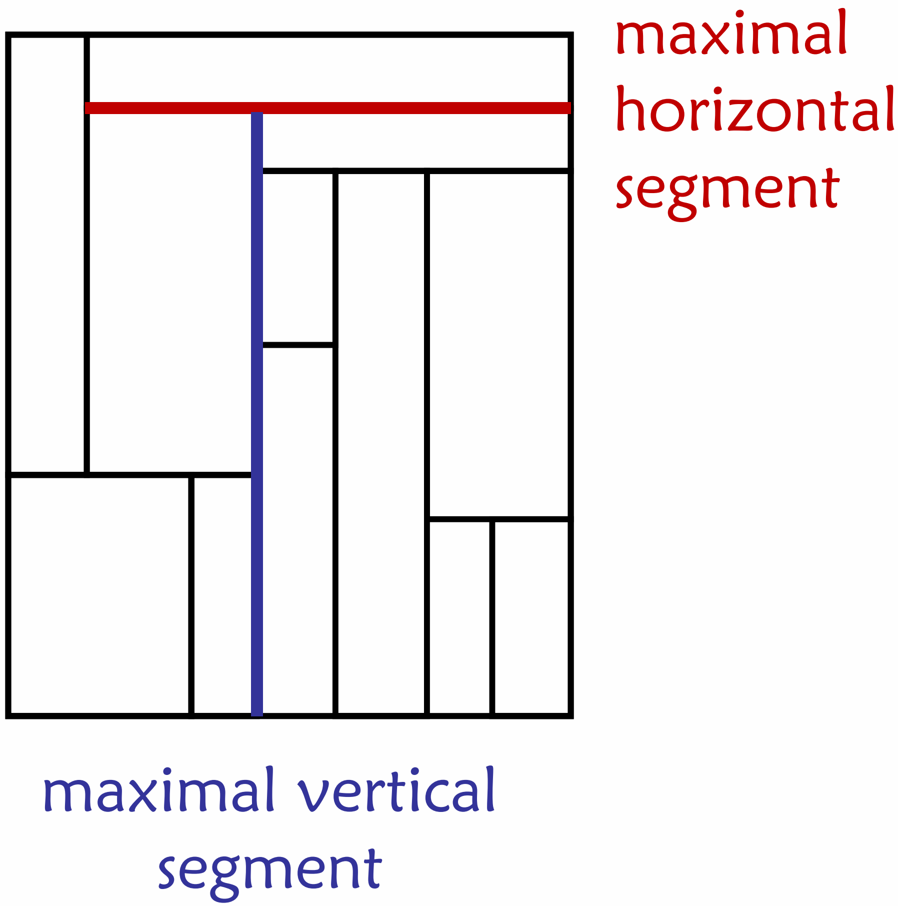
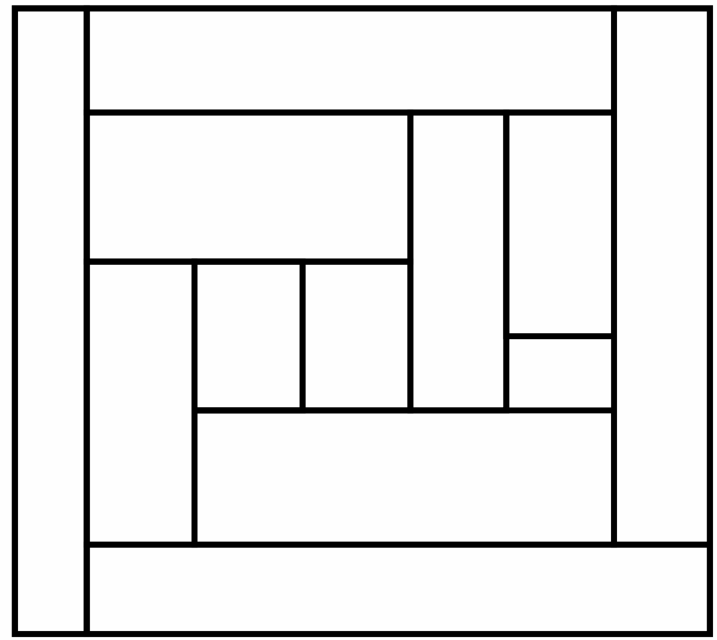
One-sided layouts

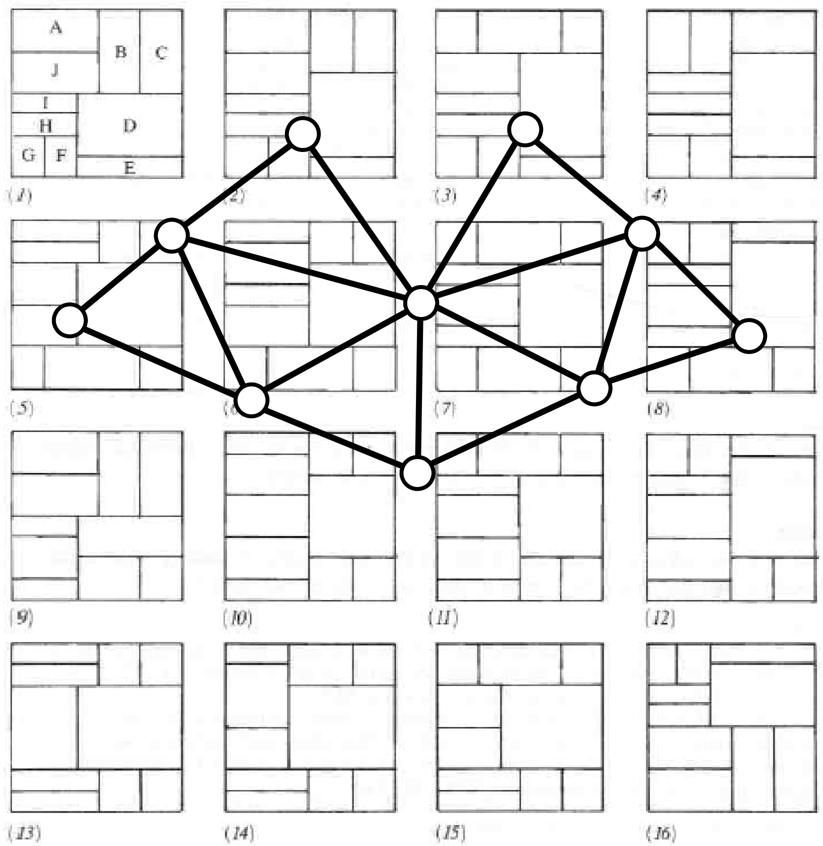




One-sided layout L: every maximal line segment of L must be the side of a least one rectangle

Finding one-sided layouts

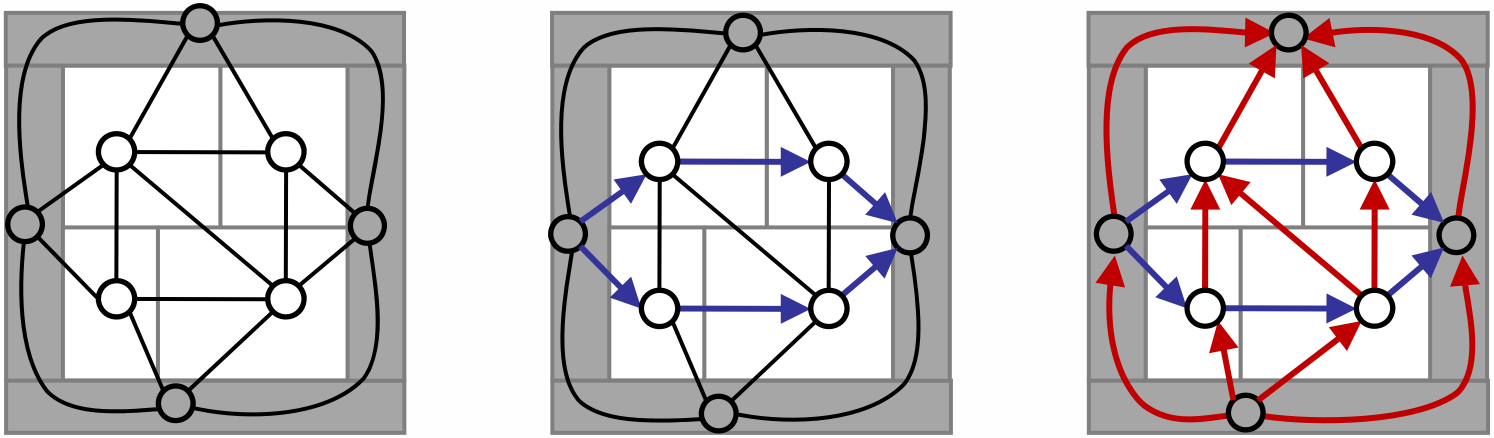
One-sided duals



[Rinsma '87]

There exists an outer-planar triangulated graph that does have rectangular duals, but no one-sided dual.

Regular edge labelings



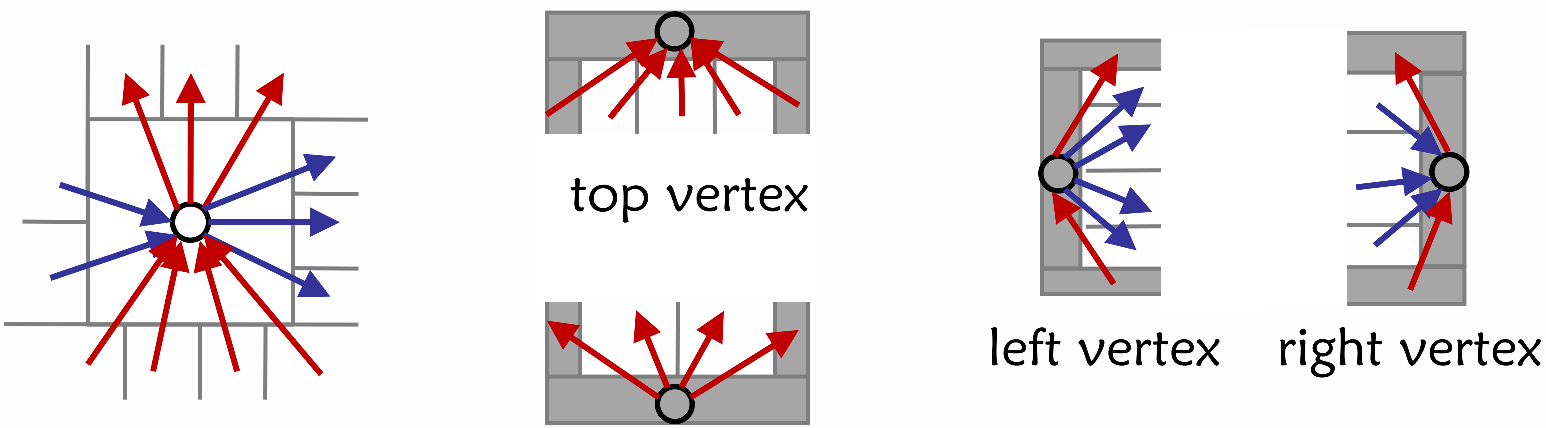
horizontal adjacency

vertical adjacency

Regular edge labeling [Kant and He'97]

inner vertex

outer vertices



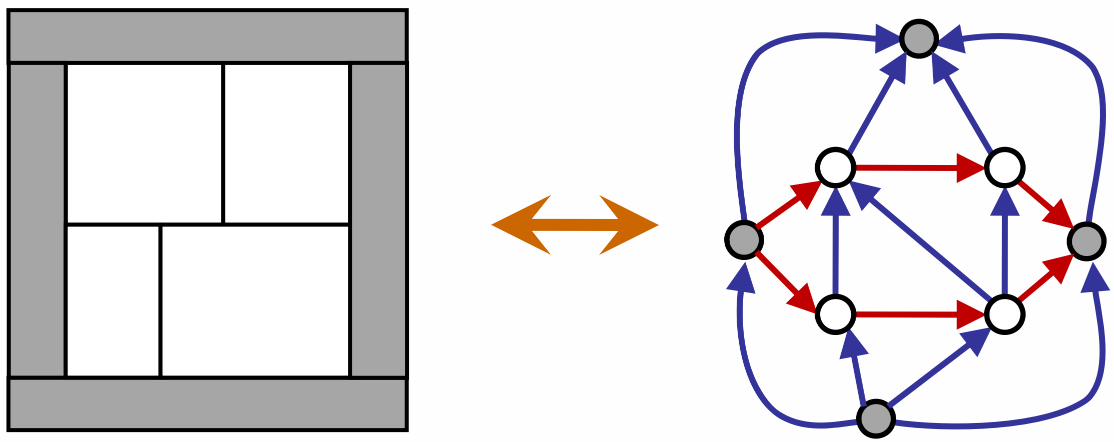
top vertex

bottom vertex

left vertex

right vertex

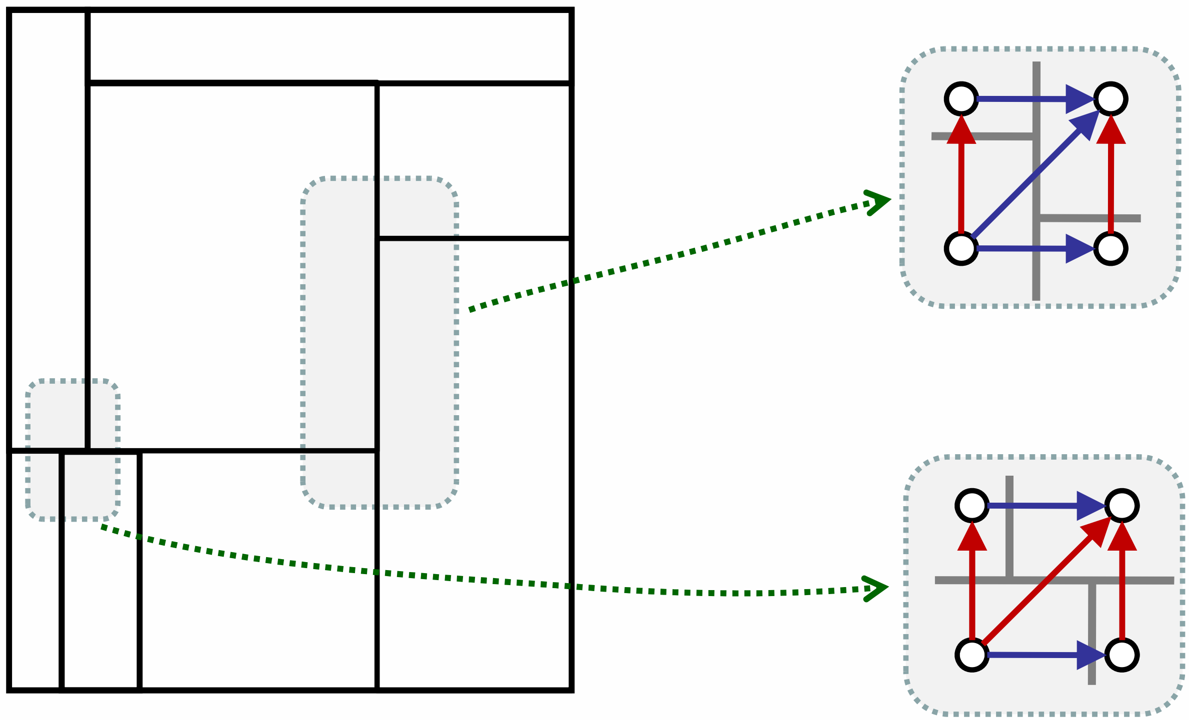
Regular edge labelings



Theorem [Kant and He'97]

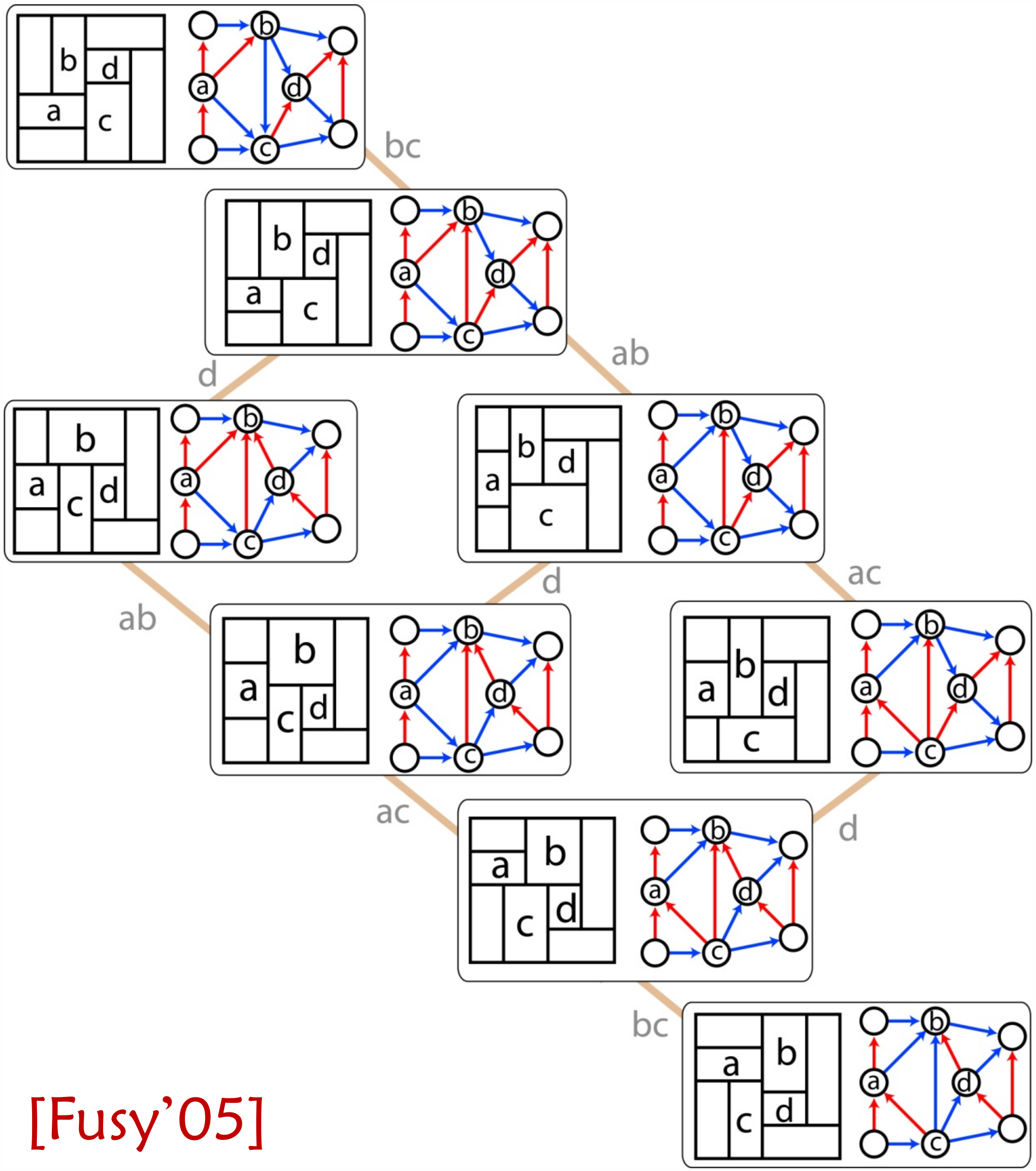
Every rectangular dual for E(G) corresponds to a regular edge labeling of E(G) and vice versa.

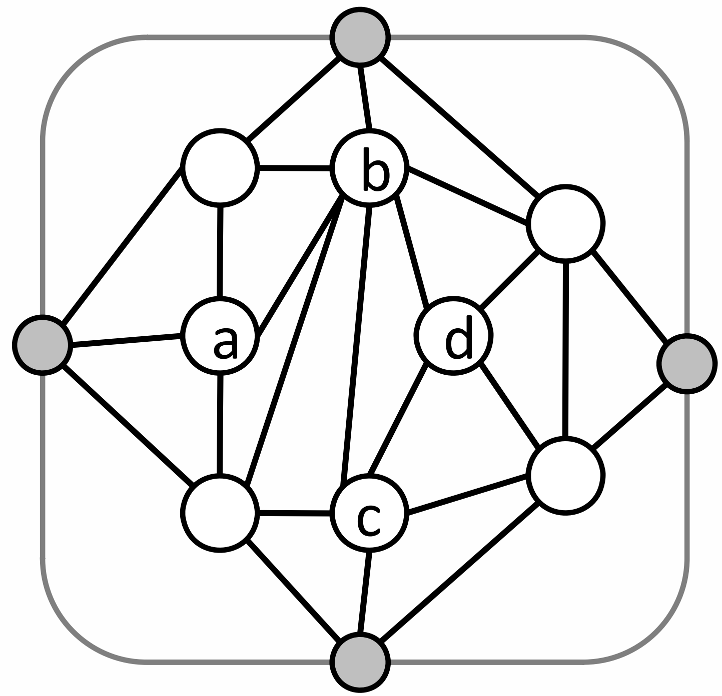
Non-one-sided layouts



Look for RELs without the patterns above

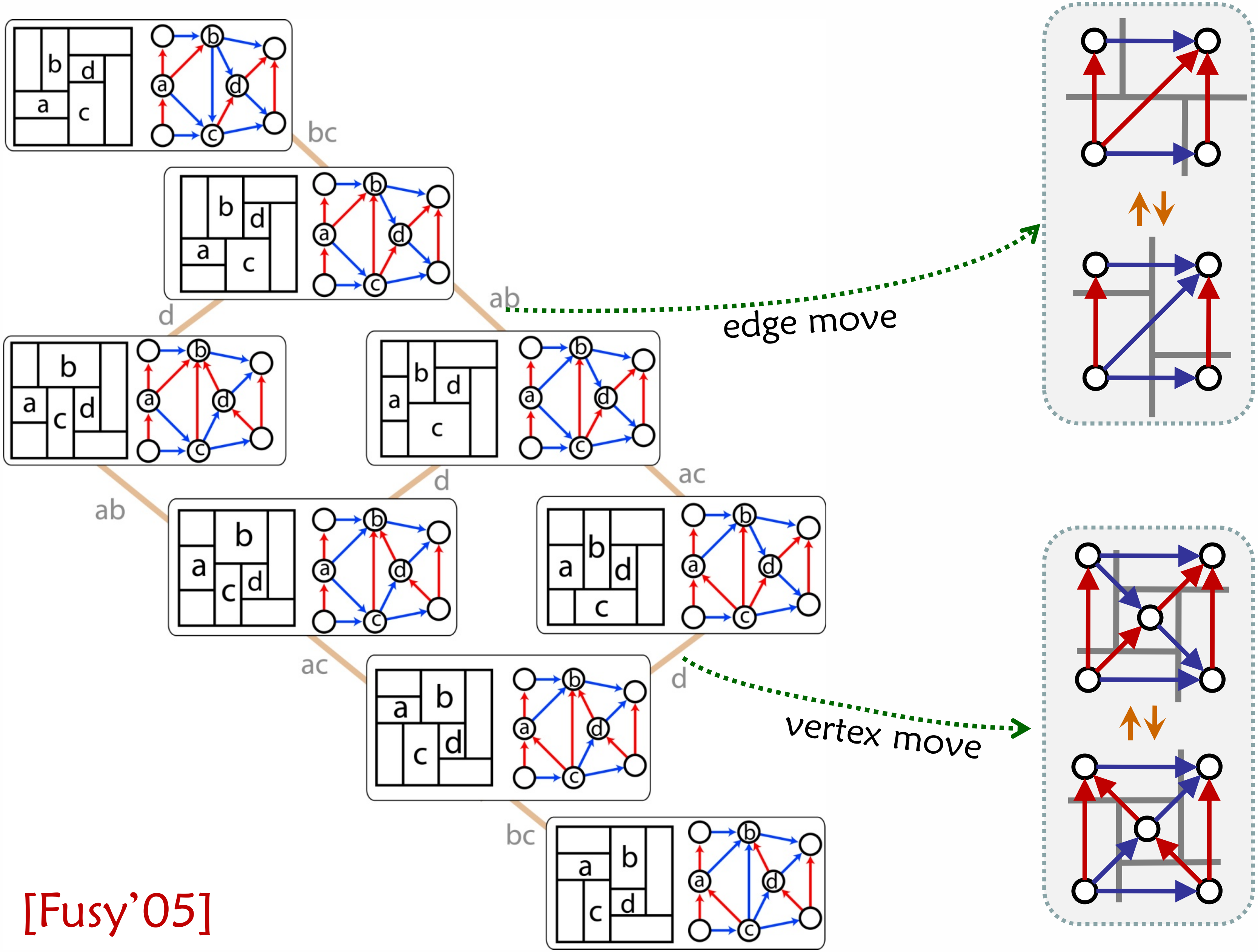
Distributive lattice of RELs





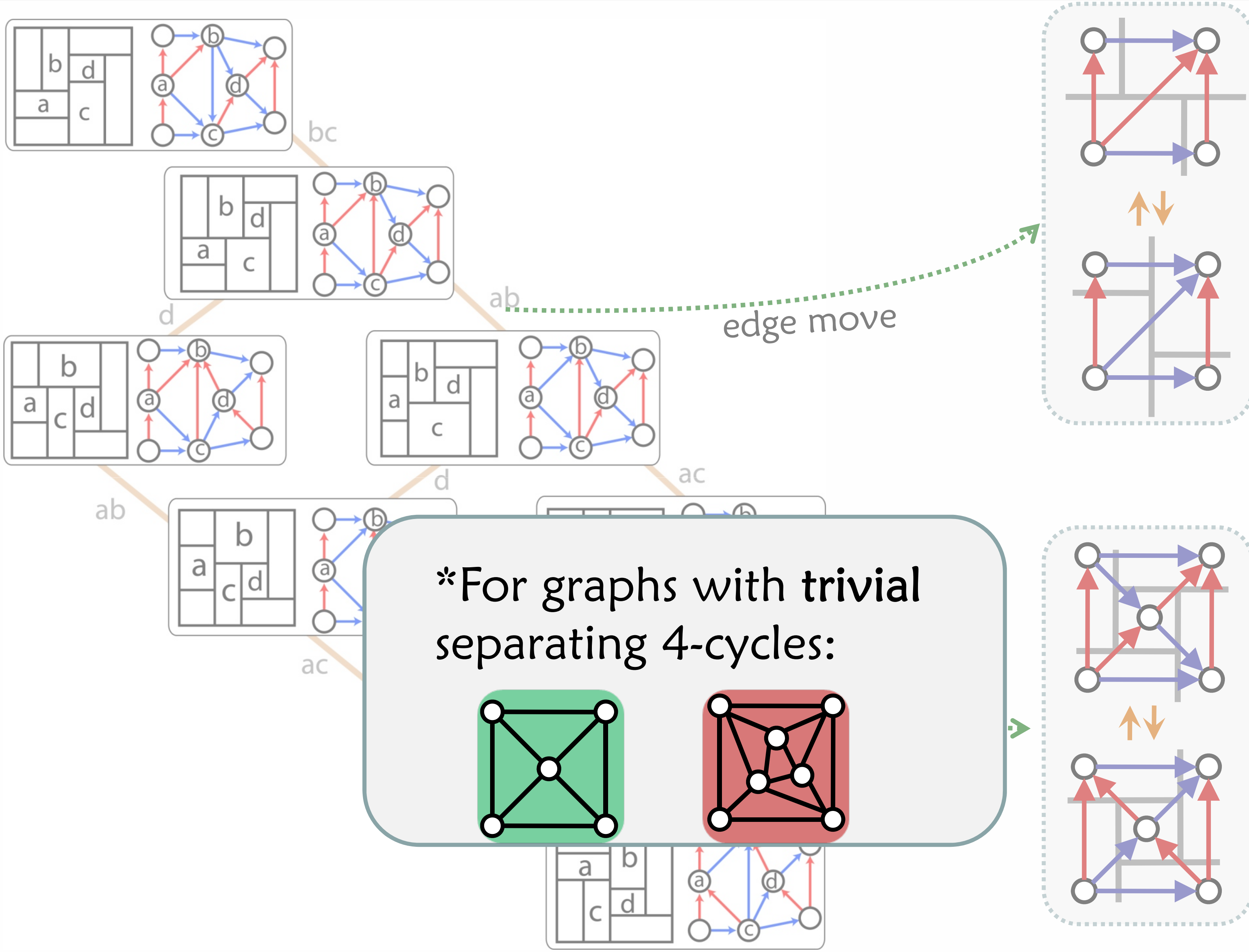
[Fusy'05]

Distributive lattice of RELs



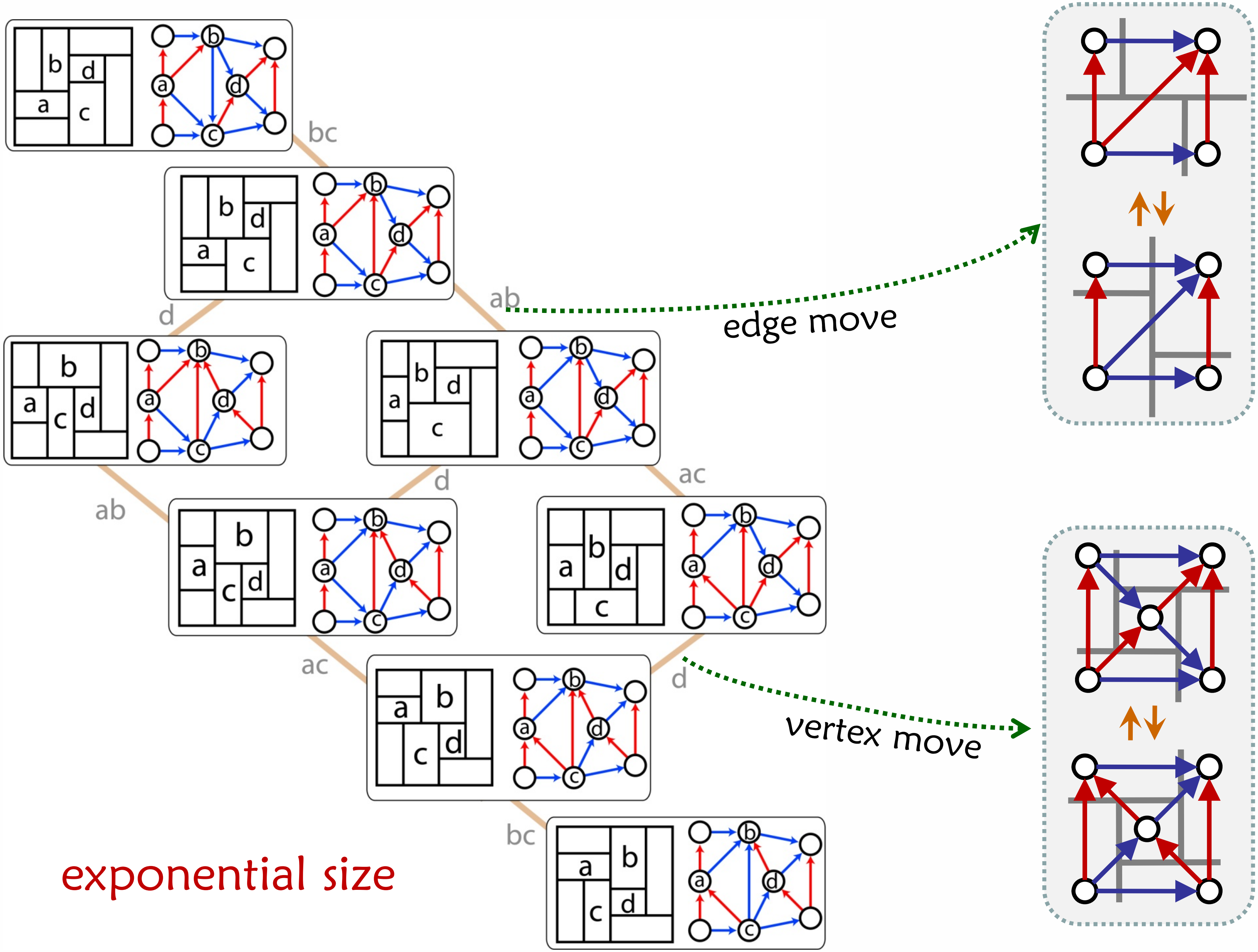
[Fusy'05]

Distributive lattice of RELs



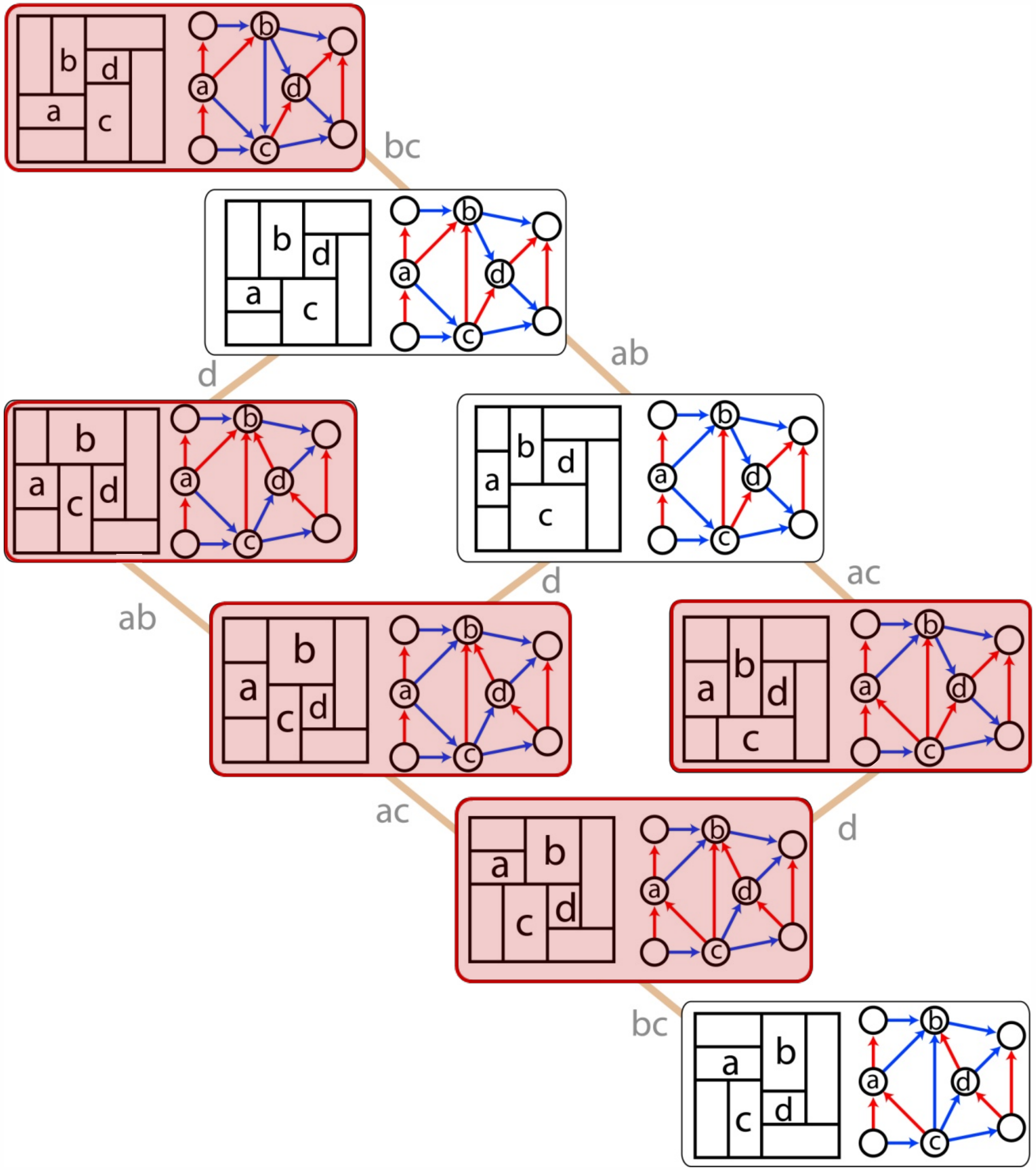
*For graphs with **trivial** separating 4-cycles:

Distributive lattice of RELs

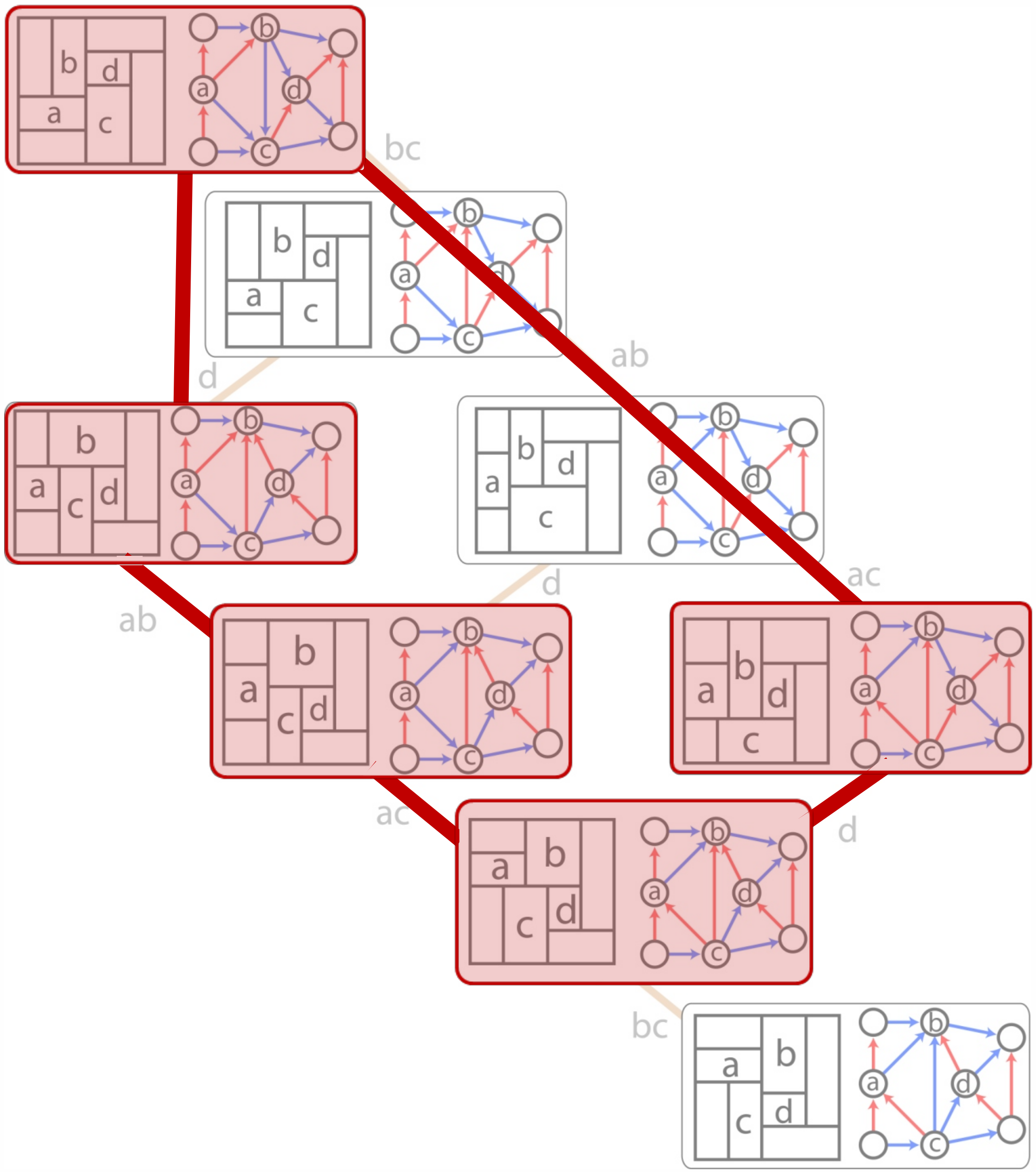


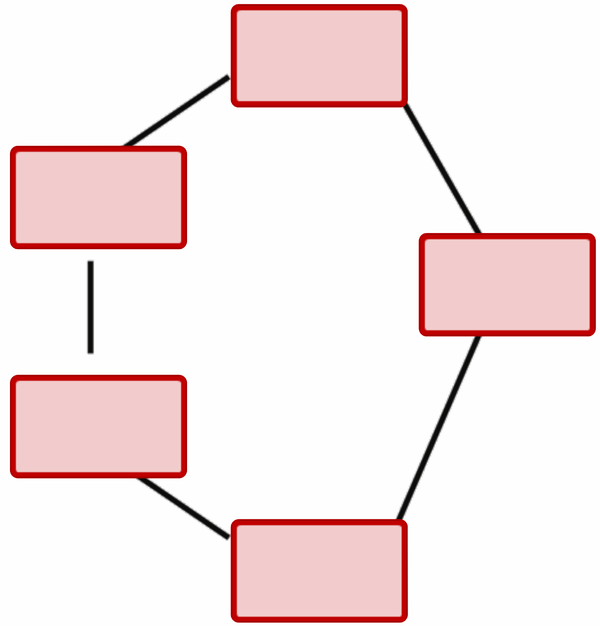
exponential size

Birkhoff's representation theorem

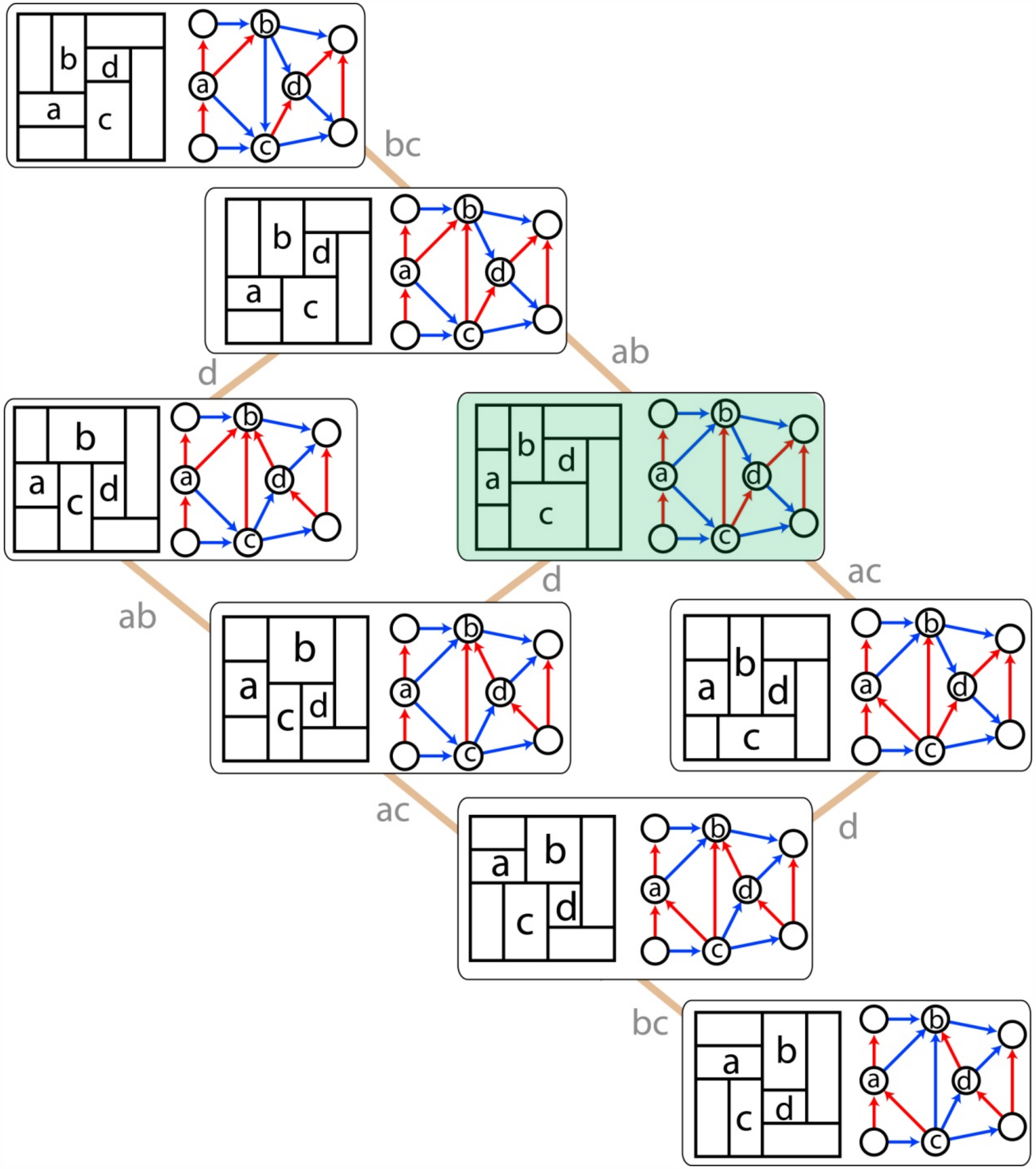


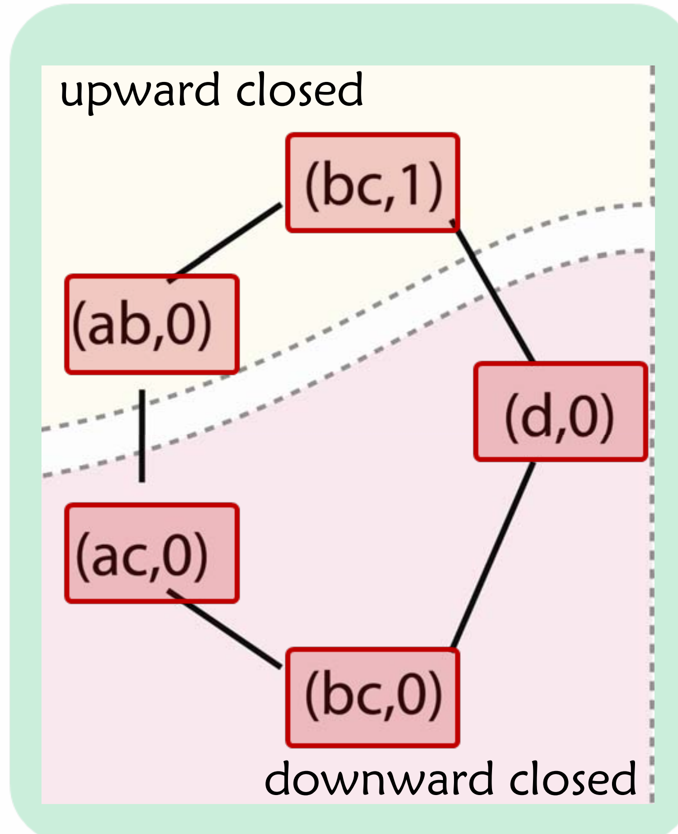
Birkhoff's representation theorem





Birkhoff's representation theorem



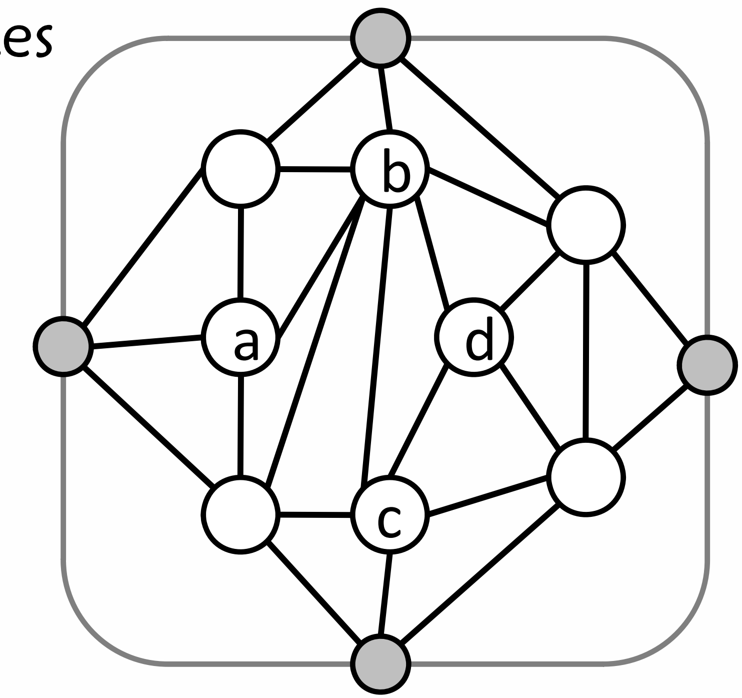


$O(n^2)$ size
can be constructed
in polynomial time

Finding area-universal layouts

- Fixed parameter tractable algorithm that runs in $O(2^{O(K^2)} n^{O(1)})$ time

K = number of degree-four vertices in the graph E(G)



Summary

Results

- We can find an area-universal layout in $O(2^{O(K^2)} n^{O(1)})$ time

- Perimeter cartograms

- Area-universal layouts for dual spanning trees in $O(n)$ time

Open problems

- Is there a polynomial algorithm for area-universal layouts?

- Can we efficiently find a layout that realizes a given area assignment in case when a graph has no area-universal layout?