

Lecture 14  
ICS 268

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Group Key Agreement  
November 17, 2001

Presented by Yongdae Kim

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Outline

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- ❖ Definitions and concepts
- ❖ Related work
- ❖ Background
- ❖ Protocols
  - Cliques
  - TGDH
  - STR
  - BD

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Background

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Group Communication Settings

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- ❖ Few-to-Many
  - Single-source broadcast: Cable/sat. TV, radio
  - Multi-source broadcast: Televised debates, GPS
- ❖ Any-to-Any
  - Collaborative applications need inherently underlying peer groups.
  - Video/Audio conferencing, collaborative workspaces, interactive chat, network games and gambling
  - Rich communication semantics, tighter control, more emphasis on reliability and **security**

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### Dynamic Peer Groups (DPG)

- ❖ Relatively small (<100 of members)
- ❖ No hierarchy
- ❖ Frequent membership changes
- ❖ Any member can be sender and receiver

My focus: key management in DPGs

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### Key Management is a building block

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### Group Key Management

- ❖ Group key: a secret quantity known only to current group members
- ❖ Group Key Distribution
  - One party generates a secret key and distributes to others.
- ❖ Group Key Agreement
  - Secret key is derived jointly by two or more parties.
  - Key is a function of information contributed by each member.
  - No party can pre-determine the result.

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### Can we use Key Distribution in DPG?

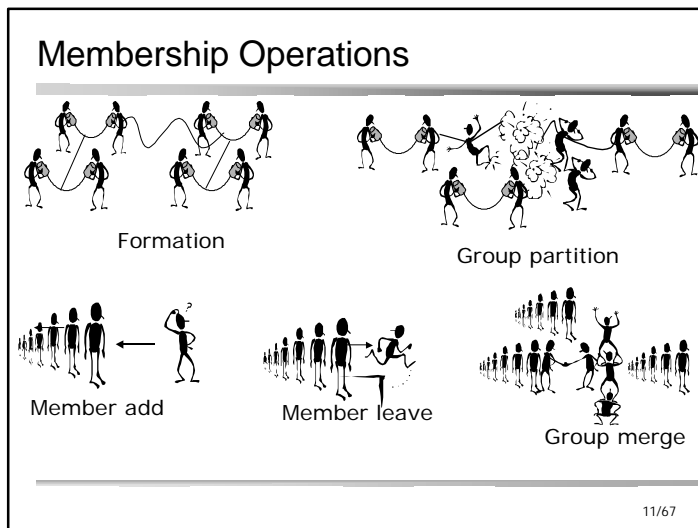
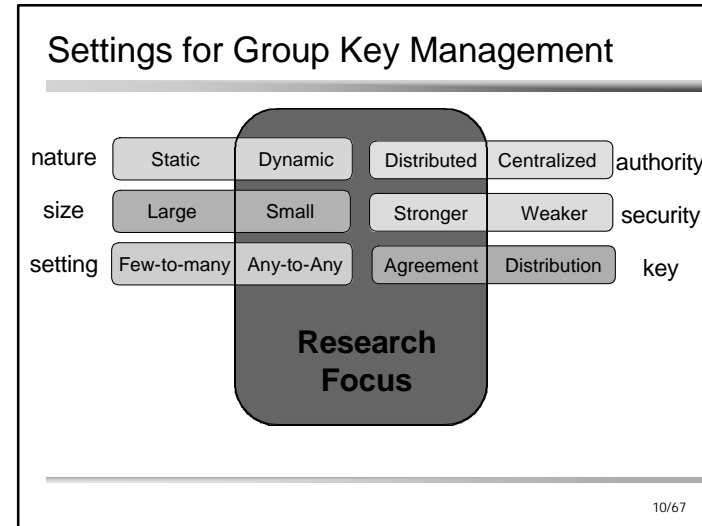
- ❖ Centralized key server
  - Single point of failure
  - Attractive attack target
- ❖ Can key server be sufficiently replicated? ⇒ Very costly
  - Availability of a key server in any and all possible partitions
    - Network can have arbitrary faults!

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### Distribution vs. Agreement

	Key Distribution	Key Agreement
Key Generation	Center	Each member's contribution
Crypto Primitive	Secret key Encryption Hash/MAC function	Extended Diffie-Hellman
Communication	Multicast or Unicast	Group communication
Computation Overhead	Small(Large for center)	Large(Similar complexity)
Group Size	> 10,000	< 100
Contributory	No	Yes
Number of round	Single	Multiple
Example	Wong and Lam OFT(McGrew, Sherman) IBM(Canetti et. al.)	BD(Burmester and Desmedt) GDH(Tsudik et. al.) TGDH(Kim et. al.) STR(Kim et. al.)

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- ### Membership Operations
- ❖ Join: a prospective member wants to join
  - ❖ Leave: a member wants to (or is forced to) leave
  - ❖ Partition: a group is split into smaller groups
    - Network failure: network event causes disconnectivity
    - Explicit partition: application decides to split the group
  - ❖ Merge: two or more groups merge to form a single group
    - Network fault heal: previously disconnected partitions reconnect
    - Explicit merge: application decides to merge multiple pre-existing groups into a single group
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## Motivation

- ❖ We need group key agreement methods satisfying the following:
  - Strong security
  - Dynamic operation
  - Robustness
  - Efficiency in communication and computation
  - Implementation, integration, and measurement

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## Why care about computation overhead?

- ❖ Most group key agreement methods rely on modular exponentiation.
  - 512 bit modular exponentiation on Pentium 400 Mhz = 2 msec
  - 1024 bit modular exponentiation = 8 msec
- ❖ Most methods require a lot of modular exponentiations for each membership operation.
  - Cliques: When current group size is  $n$ , join of a member to this group requires  $2n + 1$  modular exponentiation.

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## Security Requirements

- ❖ Group key secrecy
  - computationally infeasible for a passive adversary to discover any group key
- ❖ Backward secrecy
  - Any subset of group keys cannot be used to discover previous group keys.
- ❖ Forward secrecy
  - Any subset of group keys cannot be used to discover subsequent group keys.
- ❖ Key Independence
  - Any subset of group keys cannot be used to discover any other group keys.
  - Forward + Backward secrecy

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## Outline

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### Related Work

- ❖ Cliques
  - *Key Agreement in Dynamic Peer Groups (1996, 1997, 2000)*  
Steiner, Tsudik and Waidner  
Group Diffie-Hellman key agreement protocols  
Dynamic membership operations
  - *New Multi-party Authentication Services and Key Agreement Protocols (1998, 2000)*  
Ateniese, Steiner and Tsudik  
A notion of group key authentication is considered
  - Drawbacks  
Slow computation:  $O(n)$  computation for each membership event  
Communication overhead:  $k$  rounds for merge ( $k$ : # of new members)

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### Related Work (Continue)

- ❖ TGDH (Tree-based Group Diffie-Hellman)
  - Y. Kim, A. Perrig, G. Tsudik
  - ACM CCS 2000, Nov. 2000
  - Computation overhead reduced from  $O(n)$  to  $O(\log n)$
  - Providing robustness against cascaded failure inherently
- ❖ STR
  - Y. Kim, A. Perrig, G. Tsudik
  - IFIP SEC 2001, accepted to publication
  - Communication overhead is lower than any other methods

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### Diffie-Hellman

- ❖ Setting
  - $p$  – large prime (e.g. 512 or 1024 bits)
  - $Z_p^* = \{1, 2, \dots, p - 1\}$
  - $g$  – base generator
- ❖  $A \rightarrow B : N_A = g^{n1} \text{ mod } p$
- ❖  $B \rightarrow A : N_B = g^{n2} \text{ mod } p$
- ❖  $A : N_B^{n1} = g^{n1n2} \text{ mod } p$
- ❖  $B : N_A^{n2} = g^{n1n2} \text{ mod } p$

- ❖ Diffie-Hellman Key :  $g^{n1 n2}$
- ❖ Blinded Key of  $n1 : N_A = g^{n1} \text{ mod } p$

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### Diffie-Hellman Problem

- ❖ Computational Diffie-Hellman Assumption (CDH)
  - Loose Definition: Having known  $g^a, g^b$ , computing  $g^{ab}$  is hard.
  - CDH is not sufficient to prove that Diffie-Hellman Key can be used as secret key.
    - Eve may recover part of information with some confidence
    - One cannot simply use bits of  $g^{ab}$  as a shared key
- ❖ Decision Diffie-Hellman Assumption (DDH)
  - Loose Definition
  - Knowing  $g^a$  and  $g^b$ , and guessing  $g^c$ , can you check  $g^c = g^{ab}$ ?
  - Stronger than CDH

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### Man-in-the-Middle Attack for DH

Authentication is required

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### Authenticated Diffie-Hellman

- ❖ Implicit Authentication
  - Using Long-term Key
- ❖ Explicit Authentication
  - Using signature or MAC

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### Authenticated Diffie-Hellman

- ❖ A
  - Public Key =  $g^A$ , secret key = A (Long-term Key)
  - Computes  $K_{AB} = g^{AB}$
  - Generates a, computes  $g^a K_{AB}$
  - A  $\Rightarrow$  B:  $g^a K_{AB}$
- ❖ B
  - Public Key =  $g^B$ , secret key = B
  - Computes  $K_{AB} = g^{AB}$
  - Generates b, computes  $g^b K_{AB}$
  - Computes  $K = g^{ab} = (g^a K_{AB})^{K_{AB}^{-1} b}$
  - B  $\Rightarrow$  A:  $g^b K_{AB}$
- ❖ A can compute  $K = g^{ab} = (g^b K_{AB})^{K_{AB}^{-1} a}$

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### Background Intuition

- ❖ What should be the natural extension of Diffie-Hellman protocol to  $n$  members?
  - What will be the form of group key?
    - $g^{N_1 N_2 \dots N_n}$  where  $N_i$  is member  $i$ 's secret share
  - Which information is required to compute the group key for each member  $i$ ?
    - $g^{N_1 N_2 \dots N_n / N_i}$
  - How can we build this information?

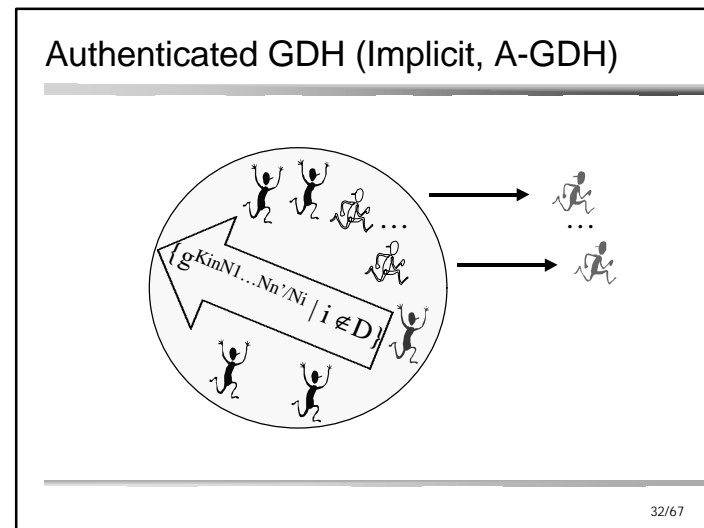
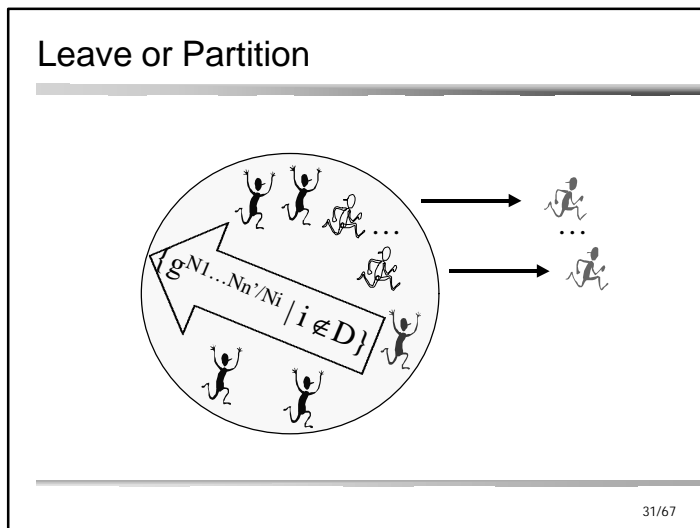
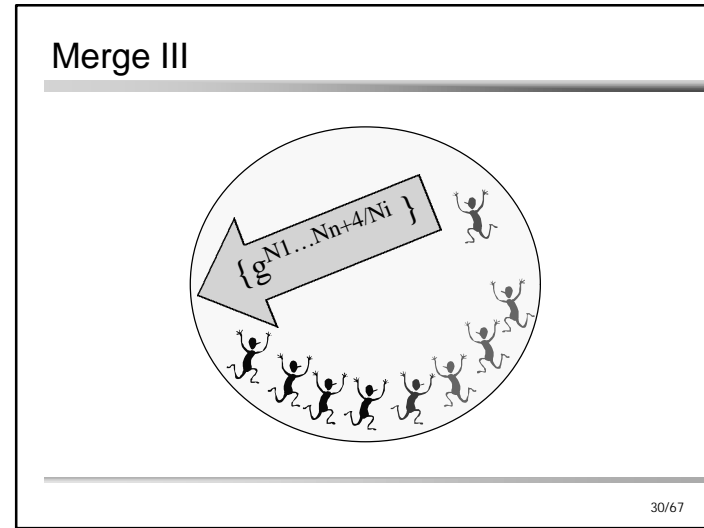
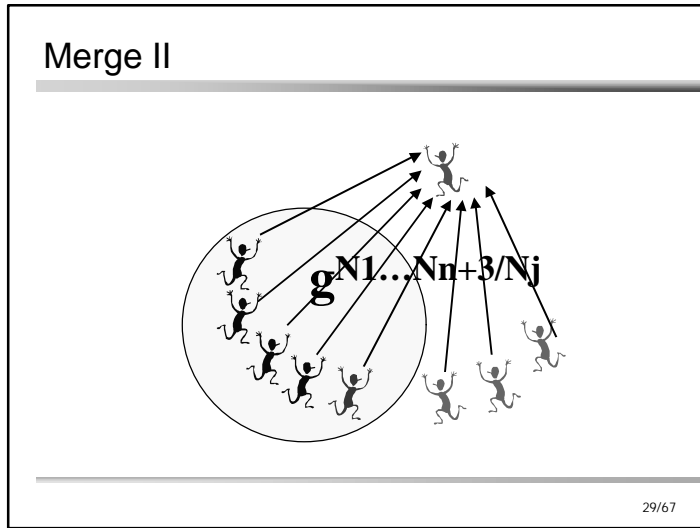
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### Joins

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### Merge I

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### Authenticated GDH (Explicit)

- ❖ Using signature or MAC
- ❖ Current Implementation uses signature

The diagram illustrates the authenticated GDH process. A group of users, represented by stick figures, is shown within a circle. A large arrow points from the group to a signature function:  $\text{Sign}(\{g^{N_1} \dots N_n / N_i \mid i \in D\})$ . Two arrows then point from this signature to individual users, indicating that the signature is used to authenticate the group's output.

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### Discussion

- ❖ Security
  - Equivalent to 2-Party Decision Diffie-Hellman problem: If we can differentiate Cliques group key with a random number, then we can differentiate 2-party Diffie-Hellman key with a random number
- ❖ Efficiency
  - $O(n)$  computation
  - $k+3$  communication round
- ❖ Robustness
  - What if a token lost?
  - Complex steps are required to achieve robustness against cascaded failure.

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### Outline

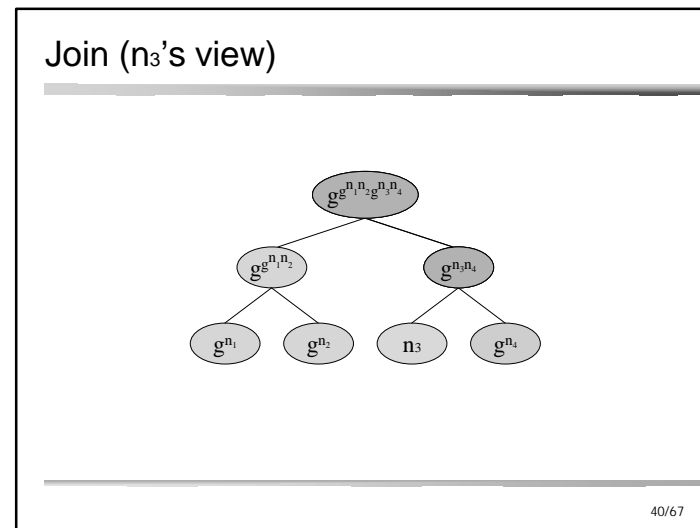
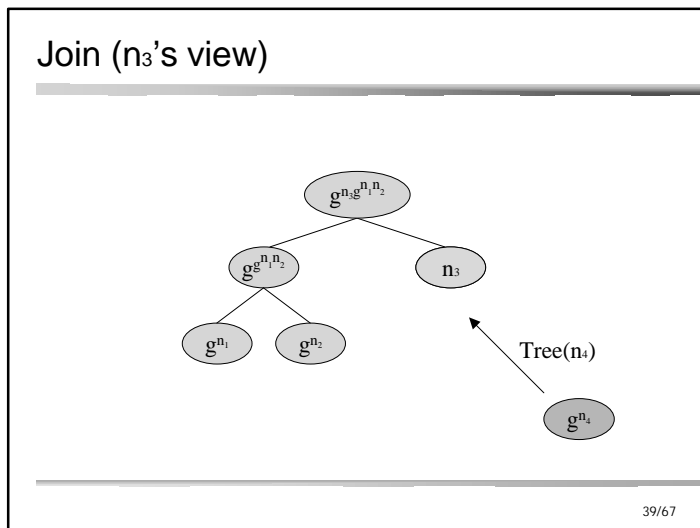
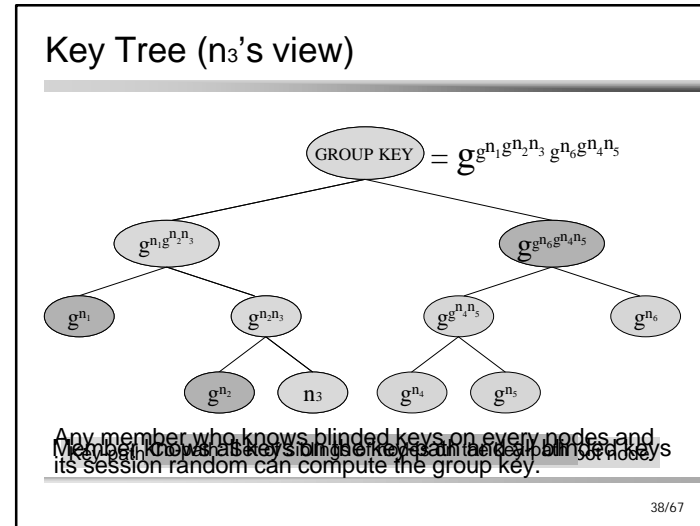
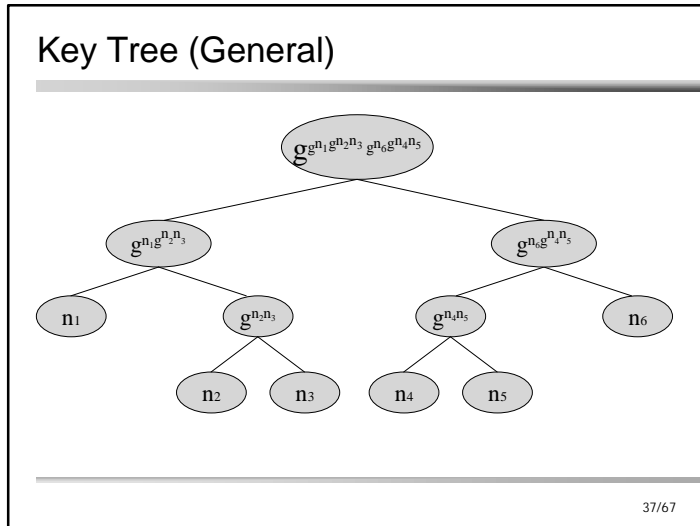
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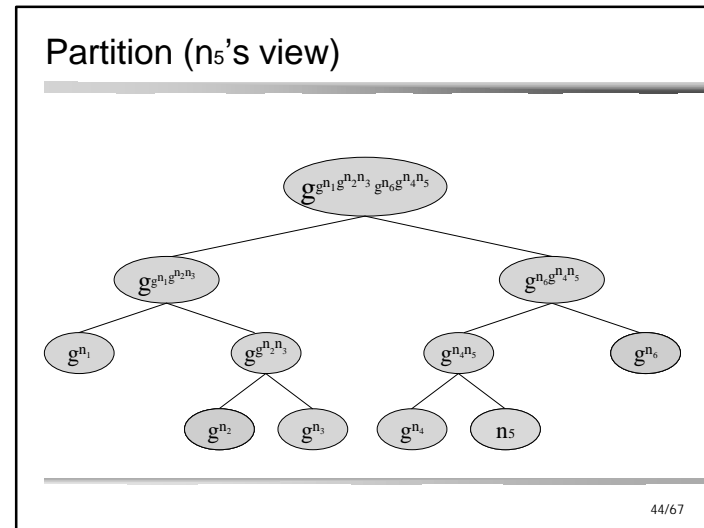
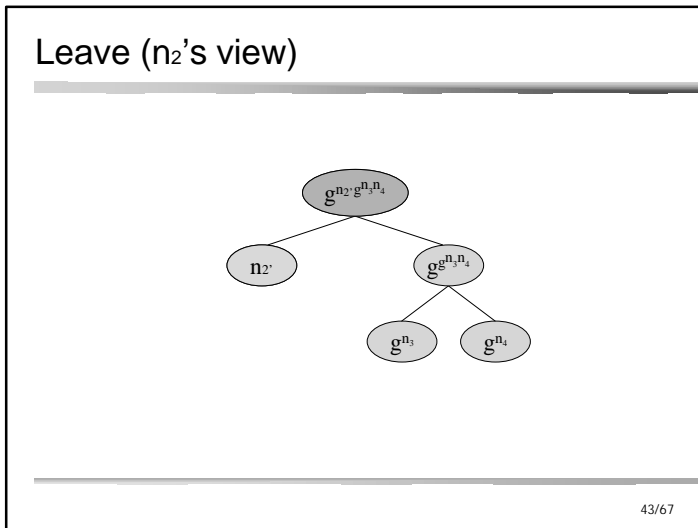
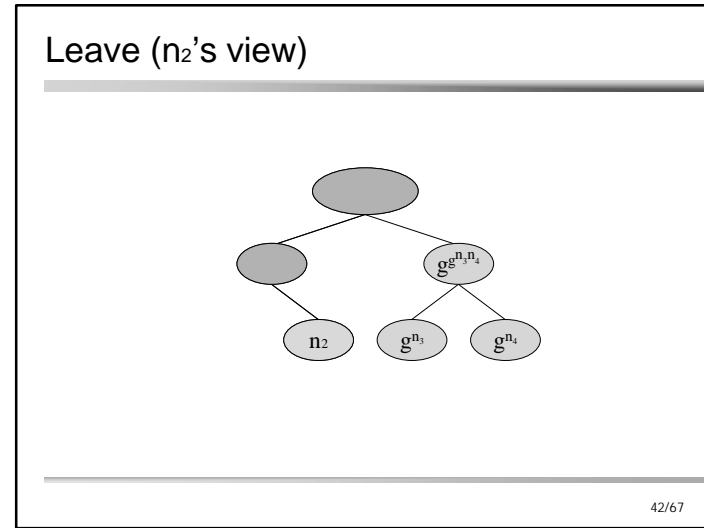
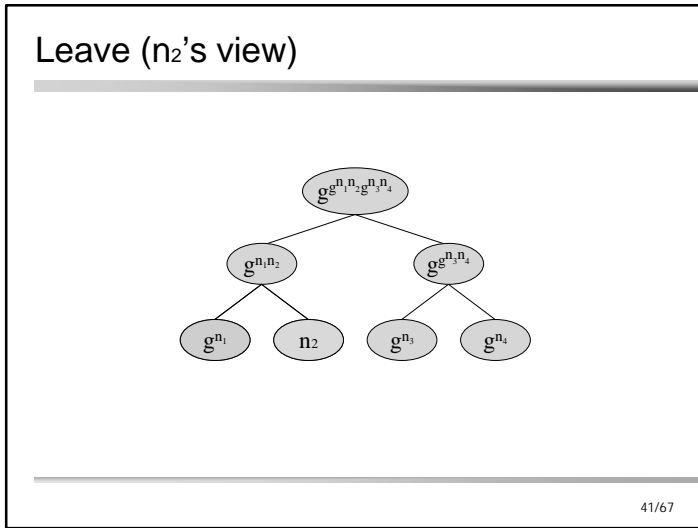
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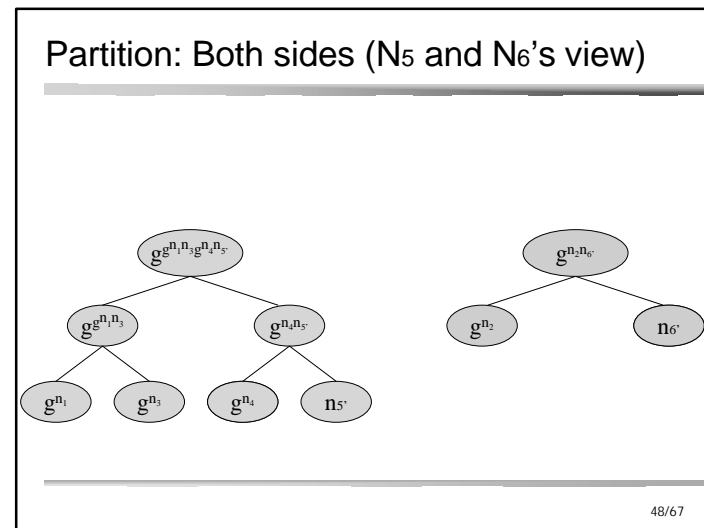
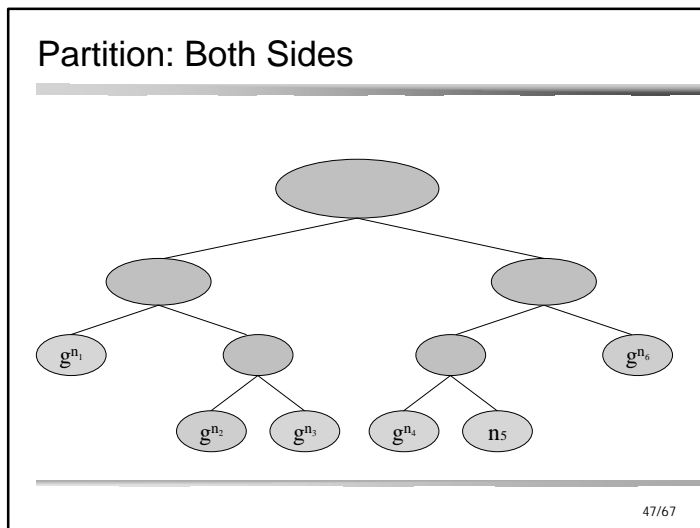
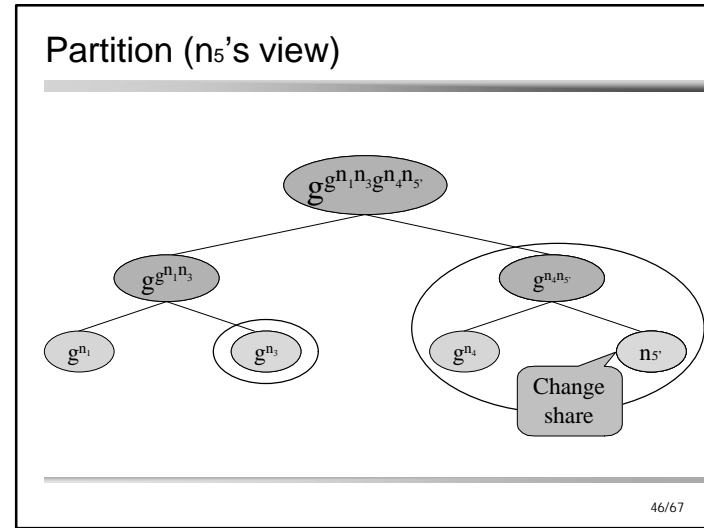
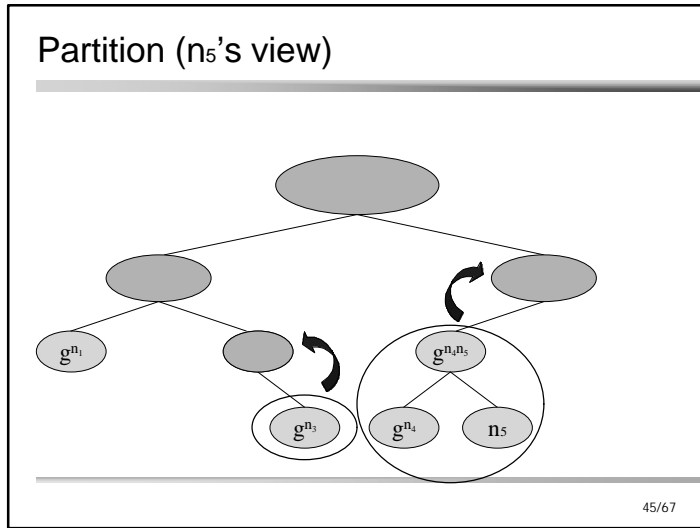
### TGDH

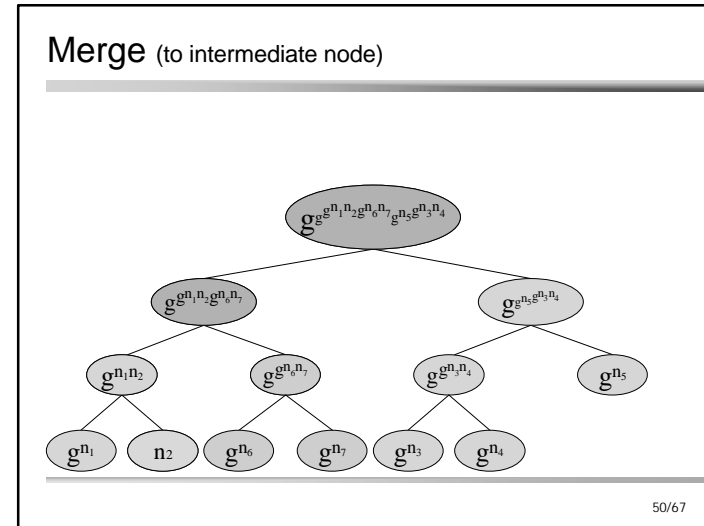
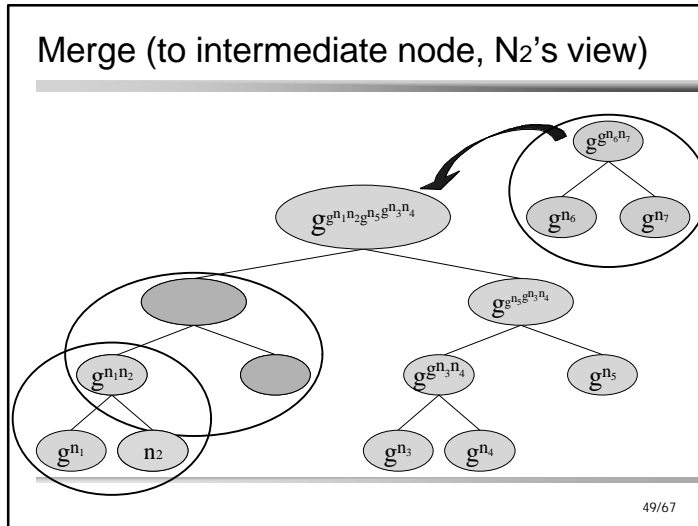
- ❖ Simple: One function is enough to implement it
- ❖ Fault-tolerant: Robust against cascaded faults
- ❖ Secure
  - Contributory
  - Provable security
  - Key independence
- ❖ Efficient
  - $d$  is the height of key tree ( $< O(\log_2 N)$ ),  $N$  is the number of users
  - Maximum number of exponentiation =  $4(d-1)$
  - # of exp. in Cliques =  $2N+1$

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- ### Tree Management: do one's best
- ❖ Join or Merge Policy
    - Join to leaf or intermediate node, if height of the tree will not increase.
    - Join to root, if height of the tree increases.
  - ❖ Leave or Partition policy
    - No one can expect who will leave or be partitioned out.
    - No policy for leave or partition event
  - ❖ Successful
    - Still maintaining logarithmic (height  $< 2 \log_2 N$ )
  - ❖ Future Work
    - Other tree management technique
    - Rebalancing
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- ### Security
- ❖ Group key secrecy
    - Intuitive Definition
      - Given all blinded keys of a random key tree, can we distinguish the group key with the random number?
  - ❖ Proof goal
    - If we can distinguish, we can distinguish 2-party DDH.
  - ❖ We can provide key independence.
    - By changing session random of a member on every additive event
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### Discussion

- ❖ Efficiency
  - Average number of mod exp:  $2 \log_2 n$
  - Maximum number of round:  $\log_2 n$
- ❖ Robustness is easily provided due to self-stabilization property

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### STR

- ❖ Communication efficient
  - Maximum 2 communication round
  - Maximum 2 broadcast messages
- ❖ Simple: One function is enough to implement it.
- ❖ Fault-tolerant: Easier than TGDH
- ❖ Secure
  - Contributory
  - Backward and forward secrecy
  - Provable security
  - Key independence
- ❖ Computation is bit more expensive.
  - Maximum number of exponentiation =  $4(N-1)$
  - N is the number of users.

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### Motivation

- ❖ Over WAN, communication is much more expensive than computation
  - Multi-round protocol is slow
- ❖ Communication always has upper bound (speed of light)
  - Computation speed increases much fast than communication
- ❖ Too many messages are also bad
  - May require retransmission

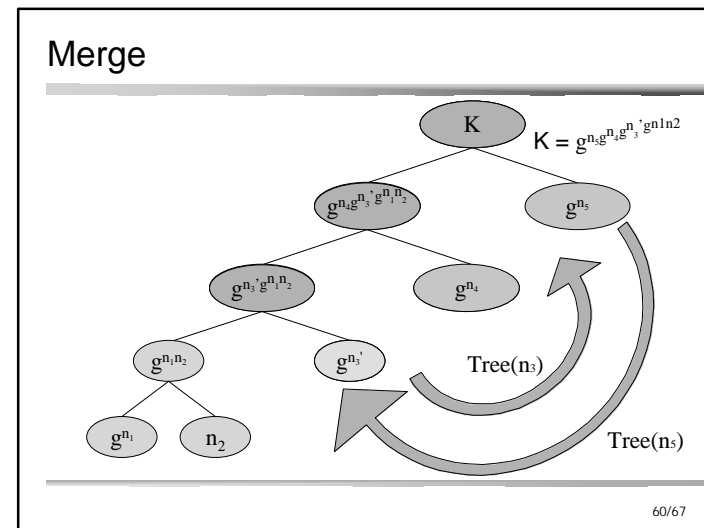
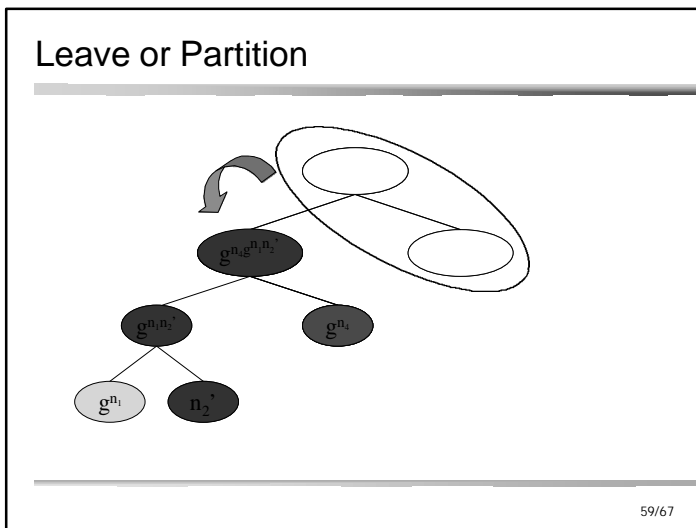
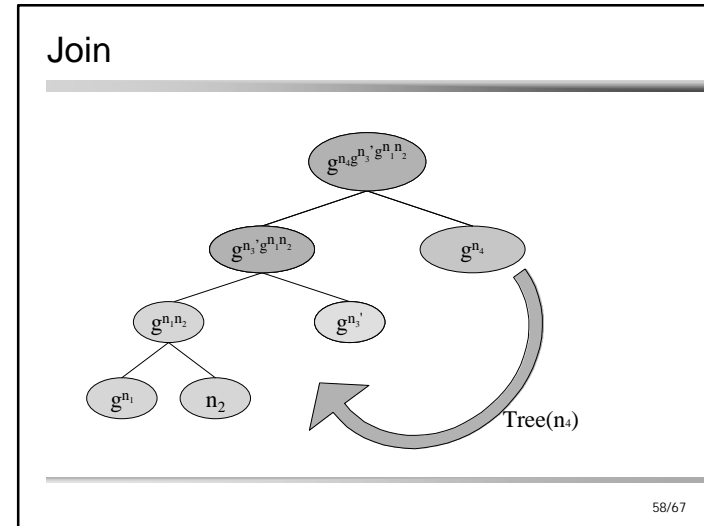
Computation(1024 DSA signature)		Communication (Ping)	
Pentium 800 Mhz	0.0037 secs	UCI ↔ Columbia univ.	0.0884 secs
Sun Ultra 250 MHz	0.0193 secs	UCI ↔ Mozambique	0.6687 secs

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### Goal

- ❖ To design a key agreement scheme which has
  - small number of round
  - Small number of message
  - But, may compute little bit more

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### Discussion

- ❖ Security
  - Same as TGDH, since STR key tree is a special case of TGDH key tree
- ❖ Efficiency
  - Average number of mod exp:  $2n$
  - Maximum number of round: 2
  - Maximum number of message: 3
- ❖ Robustness is easily provided due to self-stabilization property

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### BD

- ❖ Computation efficient
  - Constant 2 modular exponentiation
  - Constant 2 communication round
  - Each round requires  $n$  broadcasts
- ❖ No join, leave, merge, and partition protocol
  - Whenever new membership happens, need to build new group key
- ❖ Fault-tolerant
  - Whenever cascaded event happens, start from scratch
- ❖ Secure
  - Contributory
  - Backward and forward secrecy
  - Provable security
  - Key independence

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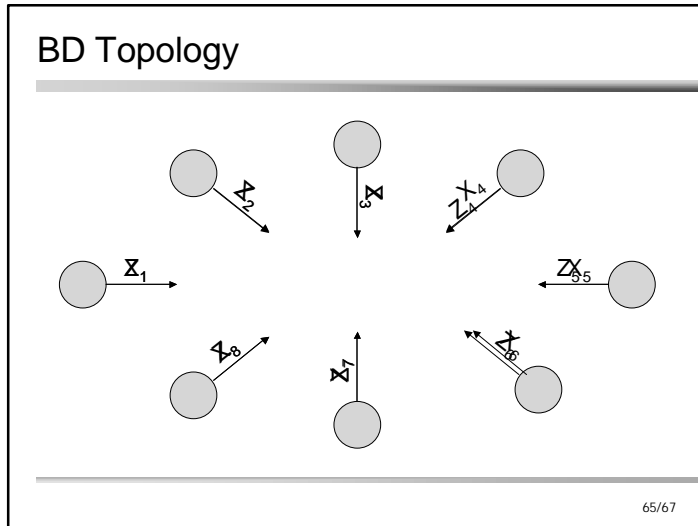
### Protocol

1. Each  $U_i$  selects random integer  $r_i$  and computes and broadcasts  $z_i = g^{r_i} \text{ mod } p$
2. Each  $U_i$  computes and broadcasts
 
$$X_i = (z_{i+1}/z_{i-1})^{r_i} \text{ mod } p$$
3. Each  $U_i$  computes the conference key
 
$$K_i = (z_{i-1})^{n r_i} X_i^{n-1} X_{i+1}^{n-2} \dots X_{i-2} \text{ mod } p$$

$$\begin{aligned}
 K_i &= (z_{i-1})^{n r_i} X_i^{n-1} X_{i+1}^{n-2} \dots X_{i-2} \\
 &= (z_{i-1})^{n r_i} (z_{i+1}/z_{i-1})^{r_i n-1} (z_{i+2}/z_i)^{r_{i+1} n-2} \dots (z_{i-1}/z_{i-3})^{r_{i-2}} \\
 &= (g^{r_{i-1}})^{n r_i} (g^{(r_{i+1} - r_{i-1})r_i})^{n-1} (g^{(r_{i+2} - r_i)r_{i+1}})^{n-2} \dots (g^{(r_{i-1} - r_{i-3})r_{i-2}}) \\
 &= g^{r_1 r_2 + r_2 r_3 + r_3 r_4 + \dots + r_n r_1}
 \end{aligned}$$

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- ### Discussion
- ❖ Security
    - Group key is indistinguishable from random number
  - ❖ Efficiency
    - Constant 2 modular exponentiation
    - Constant 2 communication round
    - Each round requires n broadcasts
  - ❖ Robustness is easily provided since we start from scratch
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### Comparison

		Comm				Comp	Robust
		Round	Msg	Uni	Broad	Exp	
CLQ	Join	4	n+3	n+1	2	n+3	Hard
	Leave, Partition	1	1	0	1	n-1	
	Merge	k+3	n+2k+1	n+2k-1	2	n+2k+1	
TGDH	Join, Merge	2	3	0	3	2log n	Easy
	Leave	1	1	0	1	log n	
	Partition	log n/2	log n	0	log n	log n	
STR	Join	2	3	1	3	7	Easy
	Leave, Partition	1	1	0	1	3n+6	
	Merge	2	3	0	3	4k+4	
BD		2	2n	0	2n	3	Easy

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