Lecture 15
November 19, 2001

- HW2
- Authentication & Identification
- Secret Sharing
- Blind signatures + e-cash

Fiat-Shamir ID Scheme

p, q – *large primes*
$n = pq$
t – *security parameter*

$Publics: n, t, ID = x^2 \bmod n$
$Secrets: p, q$ – *global*, x – *Alice*
$Note: \gcd(x, n) = 1$

1. $Alice: k \in [1, n[\ ,\ w = k^2 \bmod n$
2. $Bob: r \in [0,1]$
3. $Alice: y = kx^r \bmod n$
4. $Bob: y^2 = ? = wID^r \bmod n$

Repeat t times!

Schnorr ID Scheme

p – *large prime*
q – *large (prime) divisor of* $p-1$
g – *generator*
$a = g^{(p-1)/q} \bmod p$
t – *security parameter (40?)*

Publics : $p, q, g, a, t, ID = a^{-x} \bmod p$
Secrets : x *(Alice)*

Alice : $k \in [0, q[\,,\ w = a^k \bmod p$
Bob : $r \in [1, 2^t]$
Alice : $y = k + xr \bmod q$
Bob : $w = ? = a^y ID^r \bmod p$

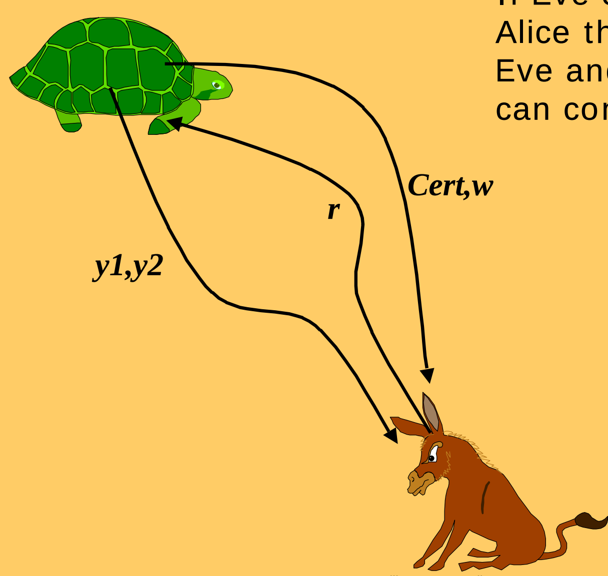
Okamoto ID Scheme

p – *large prime*
q – *large (prime) divisor of* $p-1$
g – *generator*
$a_1 = g^{(p-1)/q} \bmod p \quad a_2 = a_1^{\,c} \bmod p$
t – *security parameter*

Publics : $p, q, g, a_1, a_2, t, ID = a_1^{-x_1} a_2^{-x_2} \bmod p$
Secrets : x_1, x_2 – *Alice*, c – *global*

Alice : $k_1, k_2 \in [0, q[\,,\ w = a_1^{k_1} a_2^{k_2} \bmod p$
Bob : $r \in [1, 2^t]$
Alice : $y_1 = k_1 + x_1 r \bmod q \quad y_2 = k_2 + x_2 r \bmod q$
Bob : $w = ? = a_1^{y_1} a_2^{y_2} ID^r \bmod p$

Okamoto ID Scheme (contd)



If Eve can impersonate
Alice then
Eve and Alice
can compute c!

Guillou-Quisquater (GQ) ID Scheme

p, q – *large primes*

$n = pq$

e – global 'encryption' key

$Publics : n, e, ID = (x^{-1})^e \bmod n$

$Secrets : p, q$ – *global*, x – *Alice*

$Alice : k \in [0, n[\ ,\ w = k^e \bmod n$

$Bob : r \in [0, e[$

$Alice : y = kx^r \bmod n$

$Bob : w = ? = ID^r y^e \bmod n$

Cert = {“Alice”, ID}^{CA}

Error(s) in book!

GQ Identity-based Scheme

p,q – *large primes*

$n = pq,\ (e,d)$ – RSA key-pair

$x = (h(\text{"Alice"})^{-1})^d$

i.e., TTP signs $h(\text{"Alice"})$

Publics : $n, e, ID = h(\text{"Alice"}) \bmod n$

Secrets : p, q, d – *global*, x – *Alice*

Alice : $k \in [0, n[\ ,\ w = k^e \bmod n$

Bob : $r \in [0, e[$

Alice : $y = kx^r \bmod n$

Bob : $w = ? = ID^r y^e \bmod n$

no certificates

Converting ID to Signature Scheme (Schnorr)

p – *large prime*

q – *large (prime) divisor of* $p-1$

g – *generator*

$a = g^{(p-1)/q} \bmod p$

t – *security parameter (40?)*

Publics : $p, q, g, a, t, ID = a^{-x} \bmod p$

Secrets : x *(Alice)*

Alice : $k \in [0, q[\ ,\ w = a^k \bmod p$

Bob : $r \in [1, 2^t]$

Alice : $y = k + xr \bmod q$

Bob : $w = ? = a^y ID^r \bmod p$

p – *large prime*

q – *large (prime) divisor of* $p-1$

g – *generator*

$a \equiv 1^{1/q} \bmod p$

$t - |h()|,\ \ h() - \text{"good" hash fn}$

Publics : $p, q, g, a, t, ID = a^{-x} \bmod p$

Secrets : x

Alice : $k \in [0, q[\ ,\ w = a^k \bmod p$

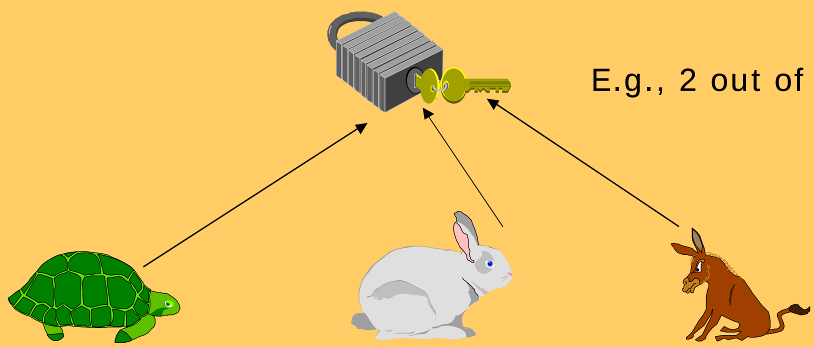
Alice : $y = k + xh(msg, w, \ldots) \bmod q$

Bob : $w = ? = a^y ID^{h(msg, w, \ldots)} \bmod p$

Secret Sharing

Why share secrets?

- Critical services: access by consent
- Replicate/backup valuable data
- In general, shared control...



E.g., 2 out of 3

Unanimous consent (t-out-of-t)

TTP is needed to generate and distribute the secret.

SETUP:
----
m – large number
TTP generates:

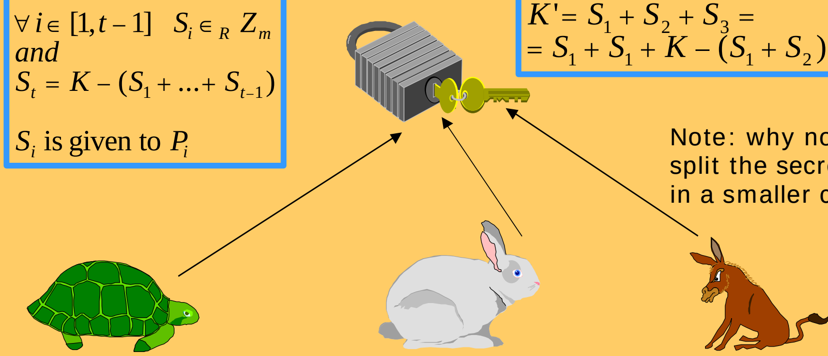
$\forall i \in [1, t-1] \quad S_i \in_R Z_m$
and
$S_t = K - (S_1 + ... + S_{t-1})$

S_i is given to P_i

RECONSTRUCTION:

Alice, Bob, Eve pool together:

$K' = S_1 + S_2 + S_3 =$
$= S_1 + S_1 + K - (S_1 + S_2) = K$



Note: why not just split the secret in a smaller chunks?

Threshold Scheme (Shamir'79) (t-out-of-n)

Need a TTP to set up the system!

SETUP:

p – large prime, $p > \max(K, n), t < n$

TTP generates:

$\forall i \in [1, n] \quad x_i \in_R Z_p$

$\forall i \in [1, t[\quad a_i \in_R Z_p$

$\forall i \in [1, n] \quad y_i = f(x_i)$ is given to P_i

where:

$f(x) = a_0 x^0 + a_1 x^1 + ... + a_{t-1} x^{t-1} \bmod p$

$a_0 = K$

publics: $\{x_1, ..., x_n\}$

secrets: $\{a_0, a_1, ..., a_{t-1}\}$

RECONSTRUCTION:

t participants pool together:

$y_1 = a_0 + a_1 x_1 + a_2 x_1^2 + ... + a_{t-1} x_1^{t-1}$

...

$y_i = a_0 + a_1 x_i + a_2 x_i^2 + ... + a_{t-1} x_i^{t-1}$

...

$y_t = a_0 + a_1 x_t + a_2 x_t^2 + ... + a_{t-1} x_t^{t-1}$

t equations, t unknowns yield

unique solution vector: $< a_0, ..., a_{t-1} >$

Shamir Threshold Schemes (example)

SETUP:

$p = 17$

$n = 5, t = 3$

TTP generates:

$x_i = i$ for all i

$a_i \in_R Z_p \ \forall i \in [1,2]$

$a_0 = K$

$y_i = f(x_i)$ is given to P_i

where:

$f(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 \bmod 17$

$y_1 = 8, y_2 = 13, y_3 = 9$

$y_4 = 5, y_5 = 11$

RECONSTRUCTION:

Suppose P_1, P_3, P_5

pool their shares

$y_1 = 8, y_3 = 10, y_5 = 11$

$a_0 + a_1 + a_2 = 8$

$a_0 + 3a_1 + 9a_2 = 10$

$a_0 + 5a_1 + 8a_2 = 11$

$a_0 = 13$

$a_1 = 10$

$a_2 = 2$

Electronic Cash

Outline

- What is electronic cash?
- Why electronic cash?
- Issues:
  - Off-line overspending
  - Anonymity
- How does e-cash work?
- Adding trustee trace-ability
- The anonymous change problem

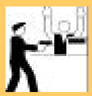
Motivation

Conventional Cash is:

•Counterfeitable

•Slow

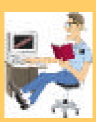
•Costly



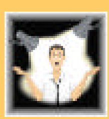
•Vulnerable

•Bad for Remote Transactions

Credit Cards, Bank Cards, Checks, and Phone/subway cards:

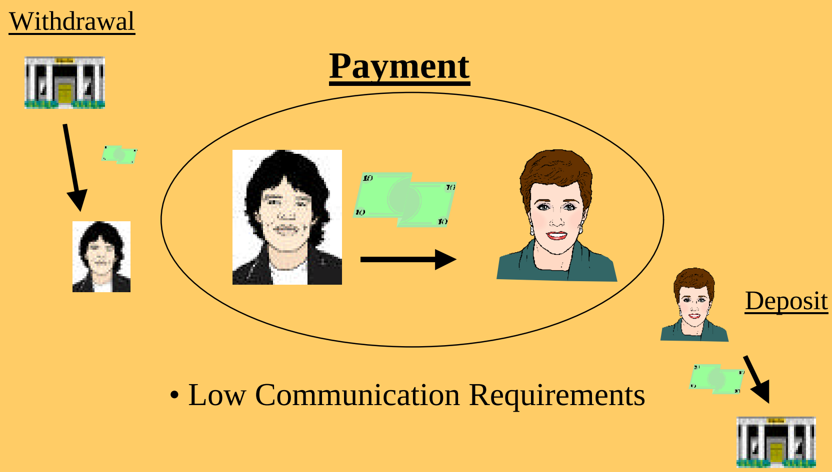


Easy Fraud



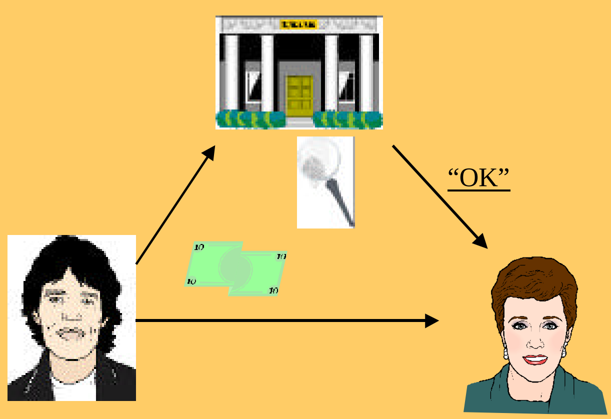
Little Privacy

Off-line Electronic Cash refers to two-party payment

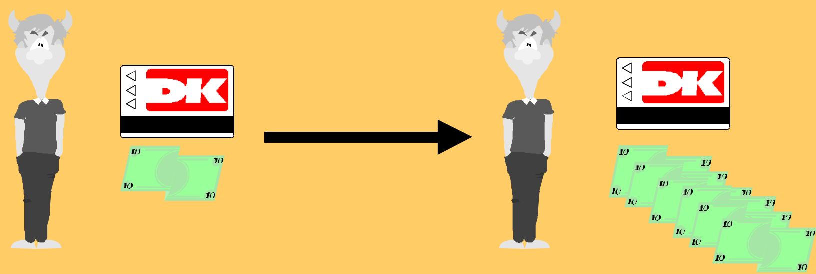


• Low Communication Requirements

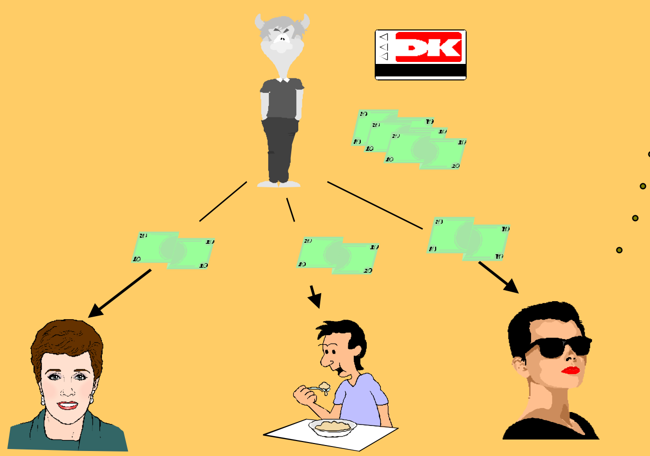
By Contrast, *On-line* Payments Look Like This



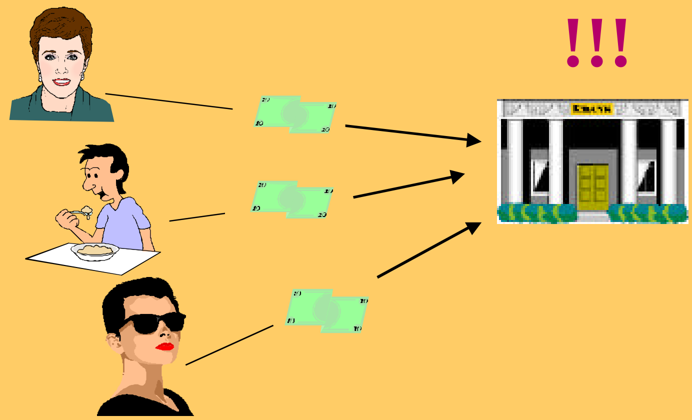
Overspending: A problem with *off-line* e-cash



Step 1: The bad user copies his money



Step 2: The bad user gives copied cash to multiple people

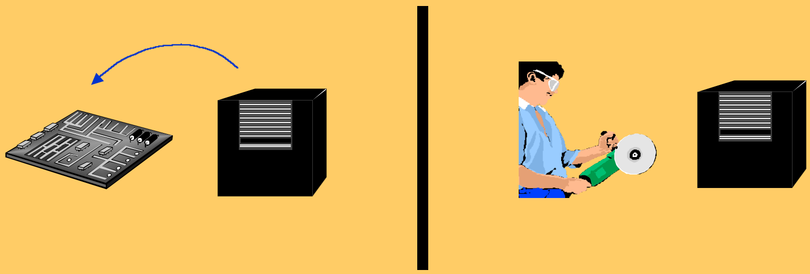


The Bank is aware of trouble only later

Techniques to Contain Over-Spending

- **Use tamper-resistant hardware to prevent over-spending (e.g., MONDEX in Europe)**
- **Trace over-spenders**
- **Blacklist over-spenders**
- **Put a bound on dollar-value of off-line transactions**

Tamper-resistance is great -- so far as it works



Resources Tradeoff