Lecture 8: October 17, 2001

- Review: Rabin's OTS
- Public key cryptography
  - RSA
  - Etc.

- HW0 due today
- HW1 due next Monday (October 22)
- Midterm: Wednesday (October 24)

Rabin's One-Time Signatures

$K_i - random \quad secret\ keys \ (0 < i < 2n)$

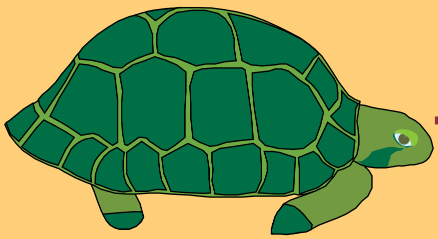
$Signature: \quad S_i = E(K_i, H(MSG))$

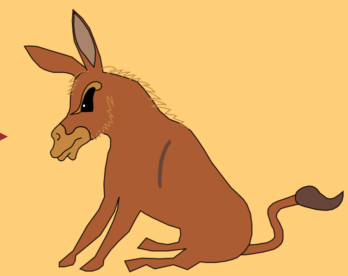
$Public-keys: \quad P_i = E(K_i, i)$

$|i| = l \quad E - block - size$

$MSG, \quad \Sigma = \{S_i \mid 0 < i < 2n\},$

$\mathrm{P} = \{P_i \mid 0 < i < 2n\},$





$Pick\ n\ random\ numbers:$

$R_i, \quad 0 < i < 2n$

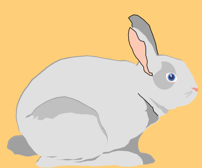
$$\Omega = \{R_i \mid 0 < i < 2n\}$$

$$\{K_{R_i} \mid R_i \in \Omega\}$$

$\forall R_i \in \Omega$

$D(K_{R_i}, S_{R_i}) \stackrel{?}{=} H(MSG)$

$D(K_{R_i}, P_{R_i}) \stackrel{?}{=} i$

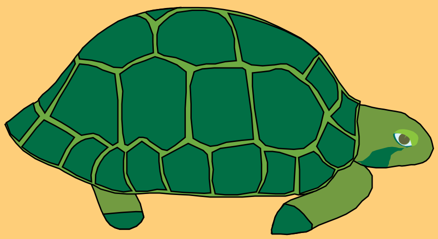


- **n – security parameter**
- **Inefficient**
- **On-line**

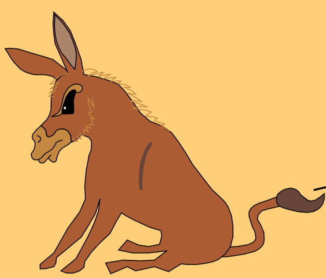
Rabin's One-Time Signatures (contd)

DISPUTES

Has P



$\{K_i \mid 0 < i < 2n\}$



$MSG, \ \Sigma$

1) $D(K_i, P_i) \stackrel{?}{=} i$

if not, Bob wins

2) $E(K_i, H(MSG)) \stackrel{?}{=} S_i$

for at most n S_i

Alice wins, else Bob wins

Note: Alice, Bob communicate with Court via authentic channels

Rationale

<u>Bob Cheats:</u>

If Bob attempts to forge Alice's signature on a new message MSG', B either:

- needs to determine at least one more key K_{n+1}

or

- determine MSG' such that H(MSG') =H(MSG).

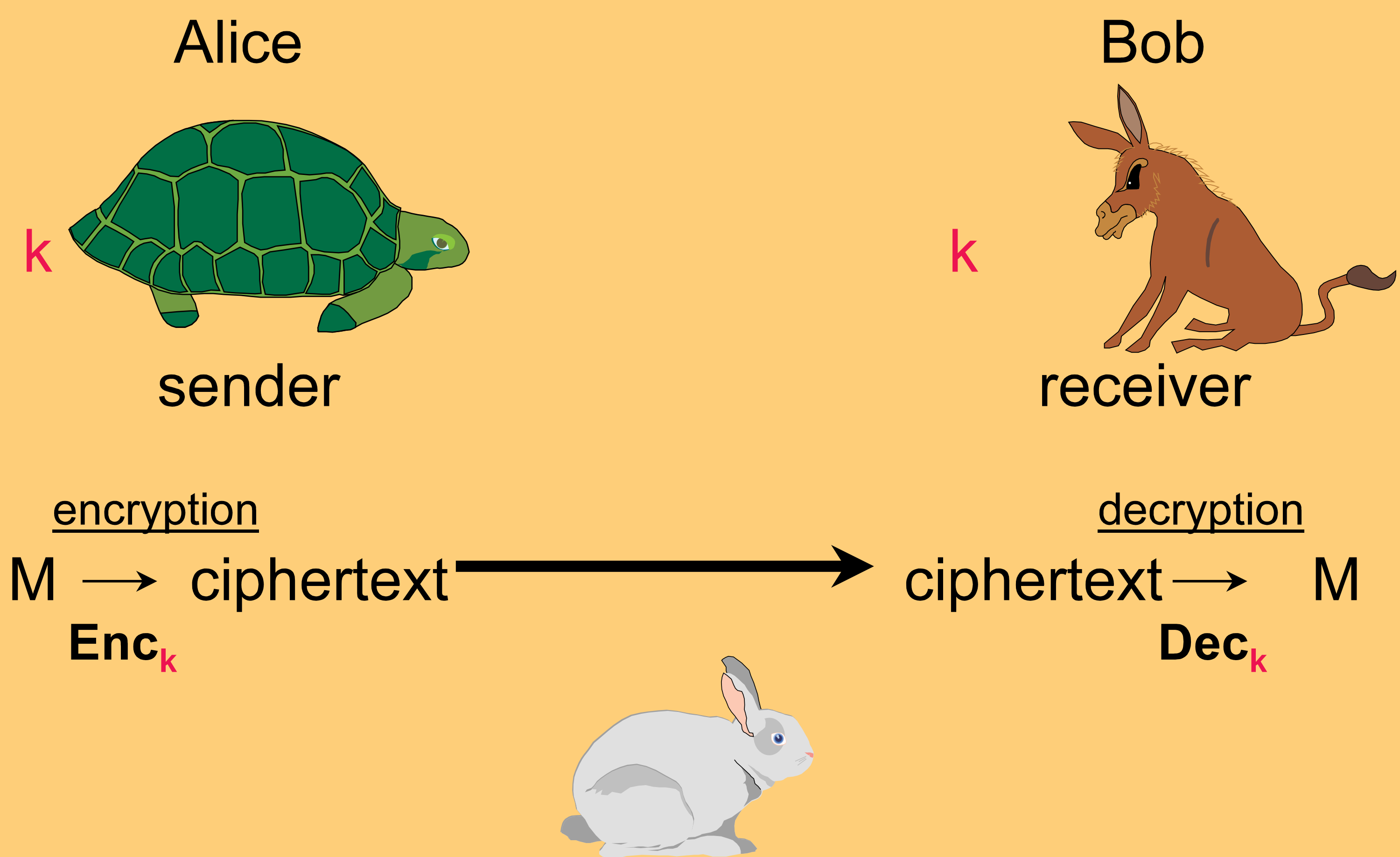
This should be infeasible if the symmetric-key algorithm E() and hash function H() are chosen appropriately.

<u>Alice Cheats:</u>

If Alice attempts to create a signature which it can later repudiate, she must ensure that precisely n S_i-s contain H(MSG) while the other n contain H(MSG') where MSG<>MSG'. She can then hope that B chooses exactly these n values in the verification procedure, the probability of which is only:

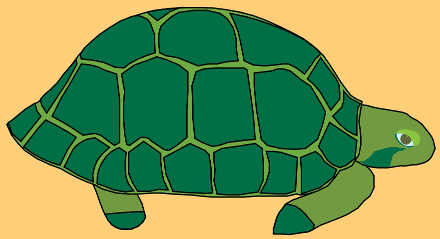
$$1/\left(\frac{2n!}{n!(2n-n)!}\right)$$

Symmetric encryption



Symmetric authentication

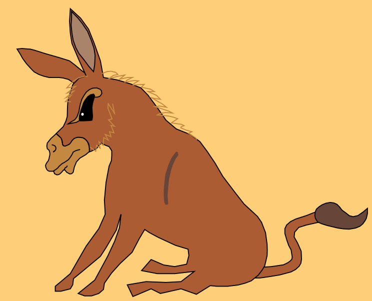
Alice

k 

sender

authentication

Bob

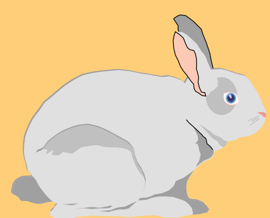
k 

receiver

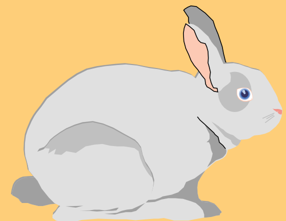
verification

$M \longrightarrow M, Auth_k(M) \longrightarrow M, Auth_k(M) \longrightarrow$ "OK"

Auth$_k$ **Verify**$_k$

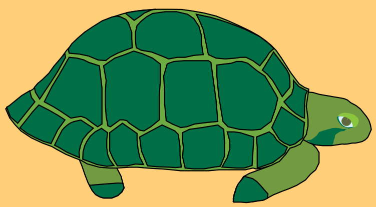


Public Key (assymmetric) encryption

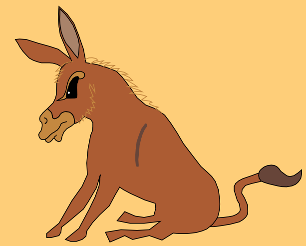


Alice | Bob

pubkey



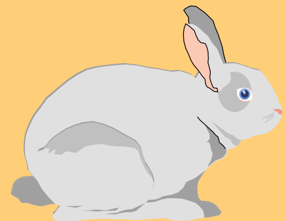
privkey



sender | receiver

$$M \xrightarrow[\textbf{Enc}_{pubkey}]{\text{encryption}} \text{ciphertext} \longrightarrow \text{ciphertext} \xrightarrow[\textbf{Dec}_{privkey}]{\text{decryption}} M$$

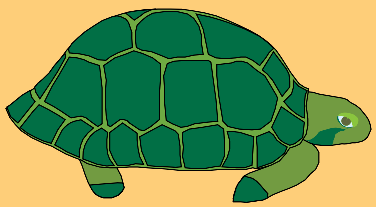
Public Key (assymmetric) authentication



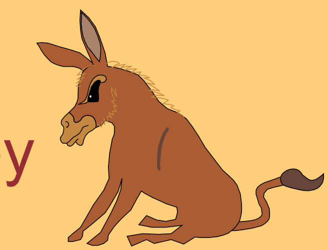
Alice

Bob

privkey



pubkey



sender

receiver

authentication

verification

$$M \longrightarrow M, Sign_{privkey}(M) \longrightarrow M, Sign_{privkey}(M) \longrightarrow \text{“OK”}$$

Sign$_{privkey}$ **Verify**$_{pubkey}$

Digital Signatures

- A public key technique to authenticate information in a way that uniquely binds the signer to the signature
- Can be used to provide non-repudiation and may be legally binding

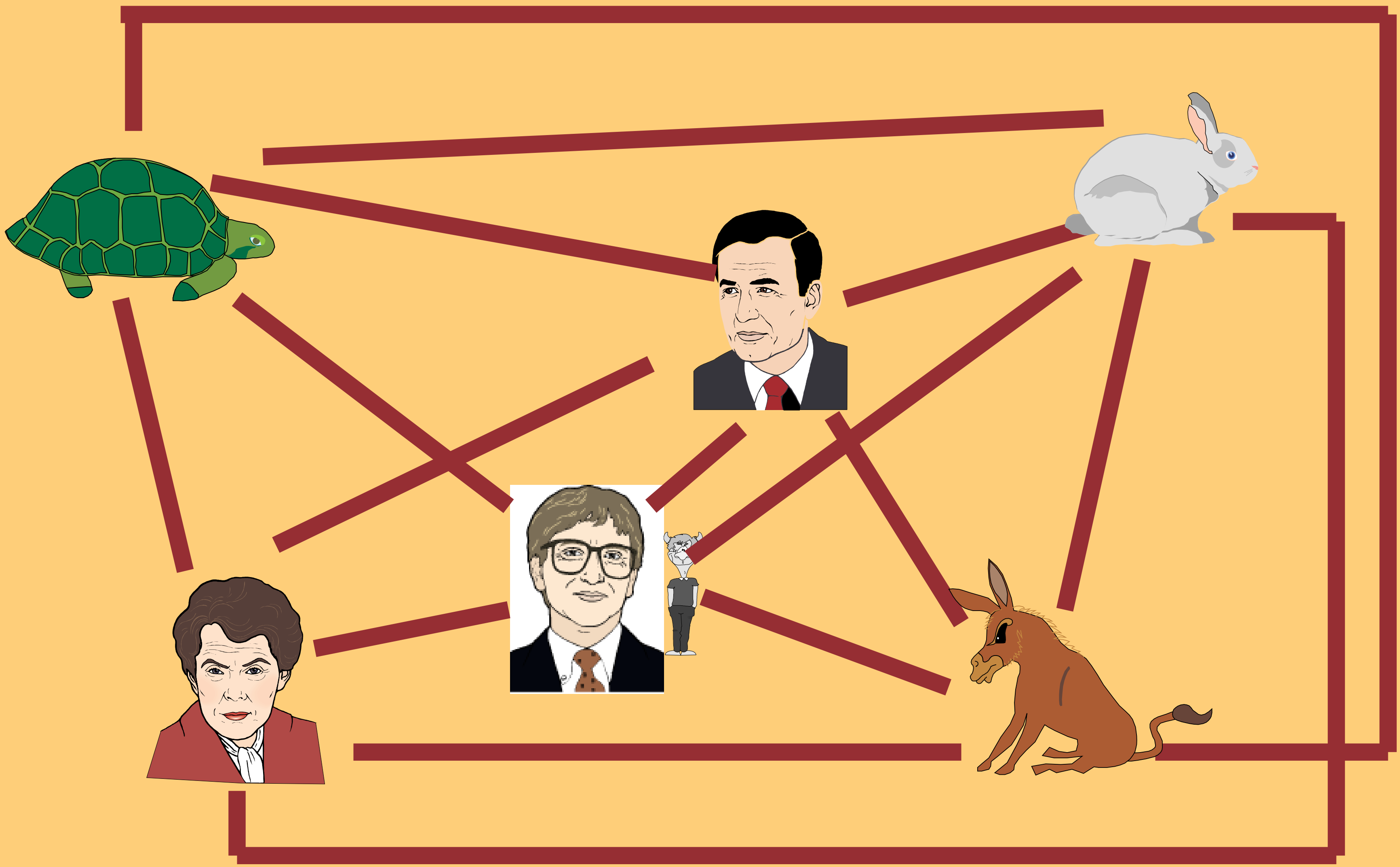
Recipient knows:

1. that the message is that of the supposed sender
2. can prove (a) to a third party

Why Public Key is important

- Reduces from N^2 to N the number of keys needed in a distributed setting (network)
- Less reliance on a “trusted center” for system availability and secrecy (e.g., electronic cash)
- Non-repudiation (Notarization)

Virtual Private Networks (VPN)



RSA (1976-8)

$$Let\ n = pq\ where\ p \neq q - (large)\ primes$$

$$e, d \in_R Z_n \ \ and\ e = d^{-1}\ and\ ed \equiv 1\ mod\ \Phi(n)$$

$$note\ that: \Phi(n) = (p-1)(q-1)$$

$$Secrets: p, q, d$$

$$Publics: n, e$$

$$Encryption: message = m$$
$$E(x) = y = m^e\ mod\ n$$

$$Decryption: ciphertext = y$$

$$D(y) = x' = y^d$$

Primality Testing

- **Needed to generate p,q**
- **Fact: primes are not that rare! N/(ln N)**
- **E.g., for 512-bit modulus, P(p-prime)=1/177**
- **So, all we do is generate 1000 or so random integers… and test them...**
- **Solovay/Strassen and Miller/Rabin:** yes-biased **Monte-Carlo composite testing algorithms.**
- **Both run in $O((\log n)^3)$ time; must be re-run x times until $[P\_error(n)]^x$ -- sufficiently small!**

Primality Testing

- **Monte Carlo algorithm**
  - **Yields yes/no answer**
  - **One is always correct**
  - **The other may be incorrect with prob.** ™
  - **Yes-biased and No-biased**
- **Las Vegas algorithm**
  - **Yields a correct answer**
  - **May not give any answer (with prob.** ™**)**

Other RSA odds and ends

- **If Eve discovers decryption exponent (d), then she can use it to factor n…=> e,d must be changed together with n!**
- **RSA decision problems are as hard as decryption!**
  - **Parity: Is last bit of cleartext 1 or 0?**
  - **Half: Is cleartext > n/2?**
  - **Let's try half(y)=0, half(E(2x))=0 => ????**

  Bit security <=> cleartext security! (see pp. 144-145)
- **RSA is** malleable**! Eve can tinker with ciphertext…**
- **For true security need randomized (probabilistic) encryption with built-in integrity/redundancy**
- **Ask me later if interested...**

Using half() to decrypt RSA

- Suppose a “black box” machine that correctly answers queries of the form:
- Assume 2-bit message: X in (00,01,10,11)
- Y=RSA(X)
- Half(Y)=1 => X_1=1
- Half(Y)=0 => X_1=0
- Half(Y*RSA(2)) mod n = 1 => => X_0=1
- Half(Y*RSA(2)) mod n = 0 => => X_0=0

Rabin PK cryptosystem (78-79)

$p <> q$ – large primes
$p, q \equiv 3 \bmod 4$
$n = pq$
$B \in [0, n[$
$P, C = Z_n$

publics – n, B
secrets – p, q

Encryption : $E_k(x) = y = x(x+B) \bmod n$
$x^2 + Bx - y = 0$

Decryption : $D_k(y) = \sqrt{B^2/4 + y} - B/2$

Let $C = B^2/4 + y$
and $X = x + B/2$
then :
$X^2 \equiv C \bmod n$
equivalently :
$X^2 \equiv C \bmod p$
$X^2 \equiv C \bmod q$
Why ?

since $p, q \equiv 3 \bmod 4$
$X = \pm C^{(p+1)/4} \bmod p$ and
$X = \pm C^{(q+1)/4} \bmod q$
then use CRT
(4times)
to find X

Using Euler's criterion

Euler's Criterion

Euler's Criterion:

If p is an odd prime, X is a quadratic residue (mod p) iff:

$$X^{(p-1)/2} \equiv 1 \bmod p$$