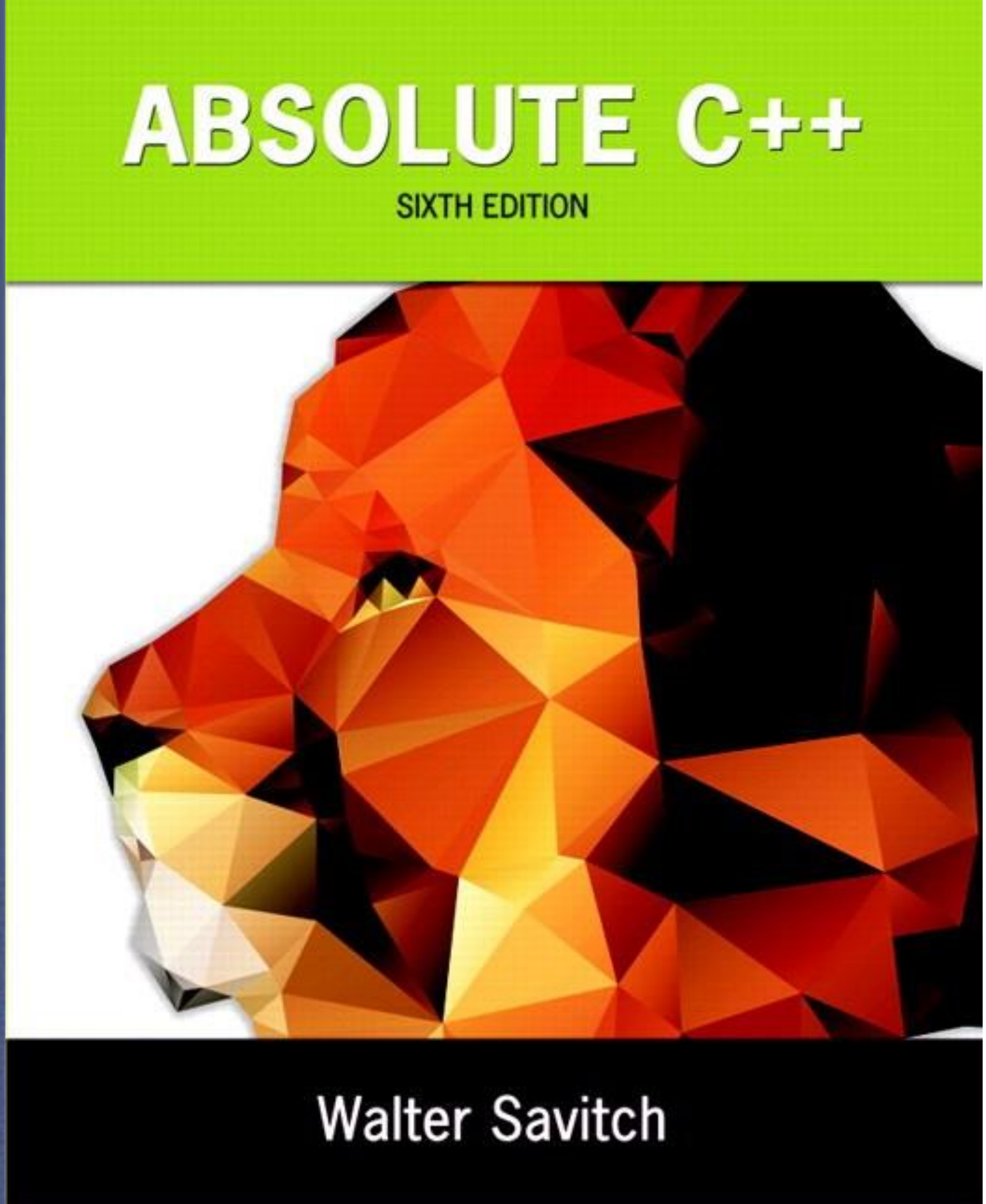


Chapter 13

Recursion

Learning Objectives

- Recursive void Functions
  – Tracing recursive calls
  – Infinite recursion, overflows
- Recursive Functions that Return a Value
  – Powers function
- Thinking Recursively
  – Recursive design techniques
  – Binary search

Introduction to Recursion

- A function that "calls itself"
  - Said to be *recursive*
  - In function definition, call to same function
- C++ allows recursion
  - As do most high-level languages
  - Can be useful programming technique
  - Has limitations

Recursive void Functions

- Divide and Conquer
  - Basic design technique
  - Break large task into subtasks
- Subtasks could be smaller versions of the original task!
  - When they are → recursion

Recursive void Function Example

- Consider task:
- Search list for a value
  - Subtask 1: search 1st half of list
  - Subtask 2: search 2nd half of list
- Subtasks are smaller versions of original task!
- When this occurs, recursive function can be used.
  - Usually results in "elegant" solution

Recursive void Function: Vertical Numbers

- Task: display digits of number vertically, one per line
- Example call:
  writeVertical(1234);
  Produces output:
  1
  2
  3
  4

Vertical Numbers: Recursive Definition

- Break problem into two cases
- Simple/base case: if n<10
  - Simply write number n to screen
- Recursive case: if n>=10, two subtasks:

  1- Output all digits except last digit

  2- Output last digit
- Example: argument 1234:
  - 1st subtask displays 1, 2, 3 vertically
  - 2nd subtask displays 4

writeVertical Function Definition

- Given previous cases:

```
void writeVertical(int n)
{
      if (n < 10)                          //Base case
            cout << n << endl;
      else
      {                                    //Recursive step
            writeVertical(n/10);
            cout << (n%10) << endl;
      }
}
```

writeVertical Trace

- Example call:
  writeVertical(123);
  → writeVertical(12);   (123/10)
      → writeVertical(1); (12/10)
          → cout << 1 << endl;
      cout << 2 << endl;
  cout << 3 << endl;
- Arrows indicate task function performs
- Notice 1st two calls call again (recursive)
- Last call (1) displays and "ends"

Recursion—A Closer Look

- Computer tracks recursive calls
  - Stops current function
  - Must know results of new recursive call before proceeding
  - Saves all information needed for current call
    - To be used later
  - Proceeds with evaluation of new recursive call
  - When THAT call is complete, returns to "outer" computation

Recursion Big Picture

- Outline of successful recursive function:
  - One or more cases where function accomplishes it’s task by:
    - Making one or more recursive calls to solve smaller versions of original task
    - Called "recursive case(s)"
  - One or more cases where function accomplishes it’s task without recursive calls
    - Called "base case(s)" or stopping case(s)

Infinite Recursion

- Base case MUST eventually be entered
- If it doesn't → infinite recursion
  - Recursive calls never end!
- Recall writeVertical example:
  - Base case happened when down to 1-digit number
  - That's when recursion stopped

Infinite Recursion Example

- Consider alternate function definition:

```
void newWriteVertical(int n)
{
    newWriteVertical(n/10);
    cout << (n%10) << endl;
}
```

- Seems "reasonable" enough
- Missing "base case"!
- Recursion never stops

Stacks for Recursion

- A stack
  - Specialized memory structure
  - Like stack of paper
    - Place new on top
    - Remove when needed from top
  - Called "last-in/first-out" memory structure
- Recursion uses stacks
  - Each recursive call placed on stack
  - When one completes, last call is removed from stack

Stack Overflow

- Size of stack limited
  - Memory is finite
- Long chain of recursive calls continually adds to stack
  - All are added before base case causes removals
- If stack attempts to grow beyond limit:
  - Stack overflow error
- Infinite recursion always causes this

Recursion Versus Iteration

- Recursion not always "necessary"
- Not even allowed in some languages
- Any task accomplished with recursion can also be done without it
  - Nonrecursive: called iterative, using loops
- Recursive:
  - Runs slower, uses more storage
  - Elegant solution; less coding

Recursive Functions that Return a Value

- Recursion not limited to void functions
- Can return value of any type
- Same technique, outline:
  1. One+ cases where value returned is computed by recursive calls
     - Should be "smaller" sub-problems
  2. One+ cases where value returned computed without recursive calls
     - Base case

Return a Value
Recursion Example: Powers

- Recall predefined function pow():
  result = pow(2.0,3.0);
  - Returns 2 raised to power 3 (8.0)
  - Takes two double arguments
  - Returns double value
- Let's write recursively
  - For simple example

Function Definition for power()

- int power(int x, int n)

```
{
    if (n<0)
    {
        cout << "Illegal argument";
        exit(1);
    }
    if (n>0)
        return (power(x, n-1)*x);
    else
        return (1);
}
```

Calling Function power()

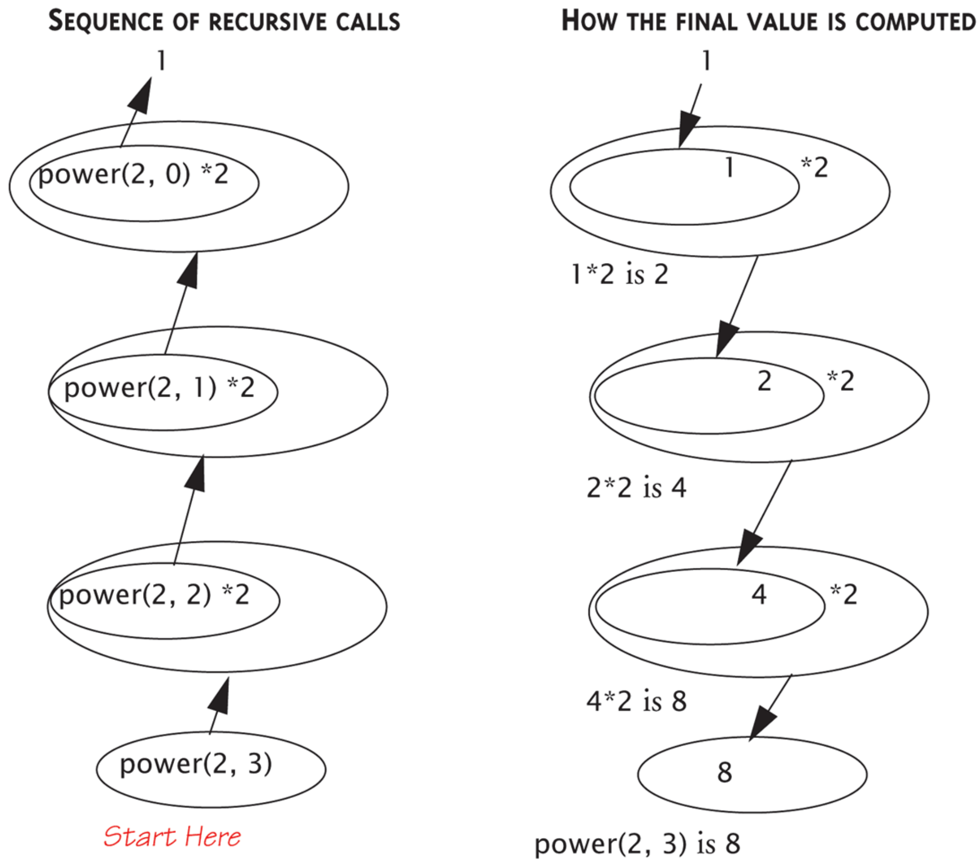
- Example calls:
- power(2, 0);
  → returns 1
- power(2, 1);
  → returns (power(2, 0) * 2);
  → returns 1
  - Value 1 multiplied by 2 & returned to original call

Calling Function power()

- Larger example:
  power(2,3);
  → power(2,2)*2
  → power(2,1)*2
  →power(2,0)*2
  →1
  - Reaches base case
  - Recursion stops
  - Values "returned back" up stack

Tracing Function power():
Display 13.4 Evaluating the Recursive Function Call power(2,3)

Display 13.4 Evaluating the Recursive Function Call power(2,3)



Thinking Recursively

- Ignore details
  - Forget how stack works
  - Forget the suspended computations
  - Yes, this is an "abstraction" principle!
  - And encapsulation principle!
- Let computer do "bookkeeping"
  - Programmer just think "big picture"

Thinking Recursively: power

- Consider power() again
- Recursive definition of power:
  power(x, n)

  returns:

  power(x, n – 1) * x
  - Just ensure "formula" correct
  - And ensure base case will be met

Recursive Design Techniques

- Don’t trace entire recursive sequence!
- Just check 3 properties:
  1. No infinite recursion
  2. Stopping cases return correct values
  3. Recursive cases return correct values

Recursive Design Check: power()

- Check power() against 3 properties:

1. No infinite recursion:
   - 2^{nd} argument decreases by 1 each call
   - Eventually must get to base case of 1
2. Stopping case returns correct value:
   - power(x,0) is base case
   - Returns 1, which is correct for x^0
3. Recursive calls correct:
   - For n>1, power(x,n) returns power(x,n-1)*x
   - Plug in values → correct

Tail recursion

- A function that is tail recursive if it has the property that no further computation occurs after the recursive call; the function immediately returns.
- Tail recursive functions can easily be converted to a more efficient iterative solution
  - May be done automatically by your compiler

Mutual Recursion

- When two or more functions call each other it is called mutual recursion
- Example
  - Determine if a string has an even or odd number of 1’s by invoking a function that keeps track if the number of 1’s seen so far is even or odd
  - Would result in stack overflow for long strings

Mutual Recursion Example (1 of 2)

```
// Recursive program to determine if a string has an even number of 1's.
#include <iostream>
#include <string>

using namespace std;

// Function prototypes
bool evenNumberOfOnes(string s);
bool oddNumberOfOnes(string s);

// If the recursive calls end here with an empty string
// then we had an even number of 1's.
bool evenNumberOfOnes(string s)
{
        if (s.length() == 0)
                return true; // Is even
        else if (s[0]=='1')
                return oddNumberOfOnes(s.substr(1));
        else
                return evenNumberOfOnes(s.substr(1));
}
```

Mutual Recursion Example (2 of 2)

```
// if the recursive calls end up here with an empty string
// then we had an odd number of 1's.
bool oddNumberOfOnes(string s)
{
        if (s.length() == 0)
                return false;        // Not even
        else if (s[0]=='1')
                return evenNumberOfOnes(s.substr(1));
        else
                return oddNumberOfOnes(s.substr(1));
}
int main()
{
        string s = "10011";

        if (evenNumberOfOnes(s))
                cout << "Even number of ones." << endl;
        else
                cout << "Odd number of ones." << endl;
        return 0;
}
```

Binary Search

- Recursive function to search array
  - Determines IF item is in list, and if so:
  - Where in list it is
- Assumes array is sorted
- Breaks list in half
  - Determines if item in 1st or 2nd half
  - Then searches again just that half
    - Recursively (of course)!

Display 13.6
Pseudocode for Binary Search

Pseudocode for Binary Search

```
int a[Some_Size_Value];
```

ALGORITHM TO SEARCH a[first] THROUGH a[last]

```
//Precondition:
//a[first]<= a[first + 1] <= a[first + 2] <=... <= a[last]
```

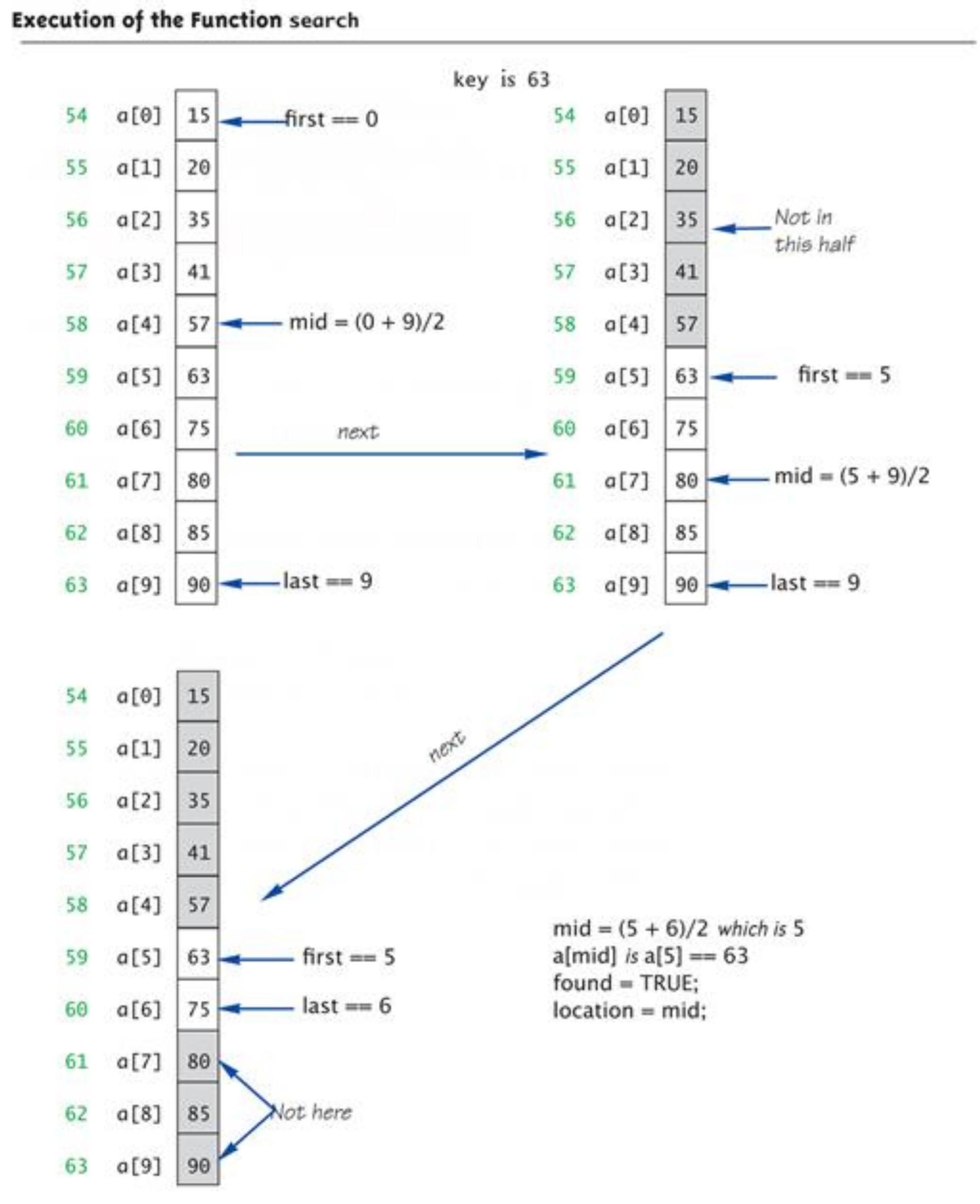
TO LOCATE THE VALUE KEY:

```
if (first > last) //A stopping case
    found = false;
else
{
    mid = approximate midpoint between first and last;
    if (key == a[mid]) //A stopping case
    {
        found = false;
        location = mid;
    }
    else if key < a[mid] //A case with recursion
        search a[first] through a[mid - 1];
    else if key > a[mid] //A case with recursion
        search a[mid + 1] through a[last];
}
```

Checking the Recursion

- Check binary search against criteria:
  1. No infinite recursion:
     - Each call increases first or decreases last
     - Eventually first will be greater than last
  2. Stopping cases perform correct action:
     - If first > last → no elements between them, so key can't be there!
     - IF key == a[mid] → correctly found!
  3. Recursive calls perform correct action
     - If key < a[mid] → key in 1st half – correct call
     - If key > a[mid] → key in 2nd half – correct call

Execution of Binary Search: **Display 13.8** Execution of the Function search



Efficiency of Binary Search

- Extremely fast
  - Compared with sequential search
- Half of array eliminated at start!
  - Then a quarter, then 1/8, etc.
  - Essentially eliminate half with each call
- Example:
  Array of 100 elements:
  - Binary search never needs more than 7 compares!
    - Logarithmic efficiency (log n)

Recursive Solutions

- Notice binary search algorithm actually solves "more general" problem
  - Original goal: design function to search an entire array
  - Our function: allows search of any interval of array
    - By specifying bounds *first* and *last*
- Very common when designing recursive functions

Summary 1

- Reduce problem into smaller instances of same problem -> recursive solution
- Recursive algorithm has two cases:
  - Base/stopping case
  - Recursive case
- Ensure no infinite recursion
- Use criteria to determine recursion correct
  - Three essential properties
- Typically solves "more general" problem