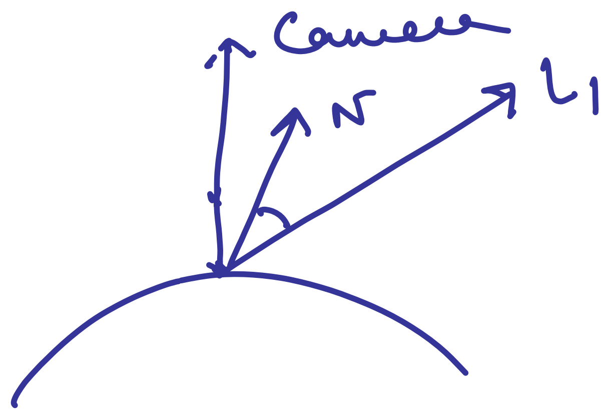
Photometric Stereo

Applicable only for Lambertian object.
As you see this is view independent.



$I_1 = \rho\, N \cdot L_1$ $\qquad$ ρ is reflectivity.

or $I_1 - \rho\,(n_x, n_y, n_z) \cdot (L_x^1, L_y^1, L_z^1) = 0$

$$I_1 - \rho \begin{bmatrix} L_x^1 & L_y^1 & L_z^1 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = 0$$

$\therefore$ with more lights

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} - \rho \begin{bmatrix} L_x^1 & L_y^1 & L_z^1 \\ L_x^2 & L_y^2 & L_z^2 \\ L_x^3 & L_y^3 & L_z^3 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = 0$$

$$\therefore \begin{bmatrix} l_x^1 & l_y^1 & l_z^1 \\ l_x^2 & l_y^2 & l_z^2 \\ l_x^3 & l_y^3 & l_z^3 \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

Also ρ is reflectivity. This is the magnitude of the normal. $\rho > 1$ indicates problem.

normalize it & you will get the surface normals.

1. Since 3 unknowns, you need minimum of three lights. More lights mean you have to solve for over constrained system.

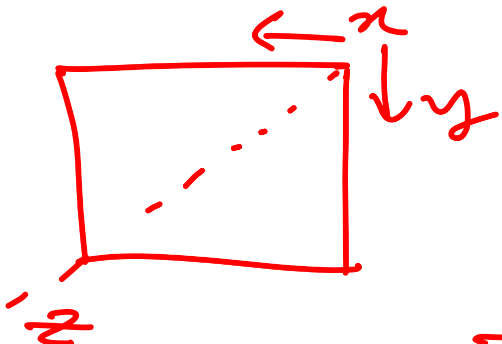
2. If the lights are coplanar, $\therefore$ the matrix is rank-deficient. So, cannot solve. $\therefore$ Lights have to be non-coplanar.

3. No depth, only surface normals. Camera does not change. $\therefore$ No issue of correspondence.

Do this computation for every pixel $\therefore$ get reflectivity & normal at every pixel.

Now note that this gives us normals but not the surface.

Assume the coordinate



Consider a hemisphere where xy plane is part of the equatorial plane.

Let the camera & light directions are on the hemisphere.

Note that z is $\perp$ xy. $\therefore$ Any normal with -ve z will be in shadow.

Assume all normals have positive z.

Let us normalize the z. $\therefore$

$$N = (f_x, f_y, 1)$$

$$\therefore \hat{N} = \left(\frac{f_x}{\sqrt{f_x^2 + f_y^2 + 1}}, \frac{f_y}{\sqrt{f_x^2 + f_y^2 + 1}}, \frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \right)$$

$\therefore$ Given (n_x, n_y, n_z)

$$f_x = \frac{n_x}{n_z}$$

$$f_y = \frac{n_y}{n_z}$$

Now we can integrate along some path to get the surface pts.

$$f(x,y) = \int_0^x f(s,y)\,ds + \int_0^y f(x,t)\,dt + c$$