Some Open Problems in Graph Theory and Computational Geometry

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Two Models of Algorithms Research

I. Read lots of theory papers

II. Choose a problem with lots of previous work (evidence it's interesting)

III. (optional) Add extra complications to the problem so you can convince people your results are more difficult than previous work

III. Find an algorithm that's better than all the previous results

IV. Write it up and publish it in theory conferences and journals

I. Learn about areas outside of theoretical CS

II. Choose a problem in one of those application areas where faster or more accurate solutions can make a practical difference

III. Abstract essential features to get new clean theoretical problem

IV. Find an algorithm that's better than all the previous results

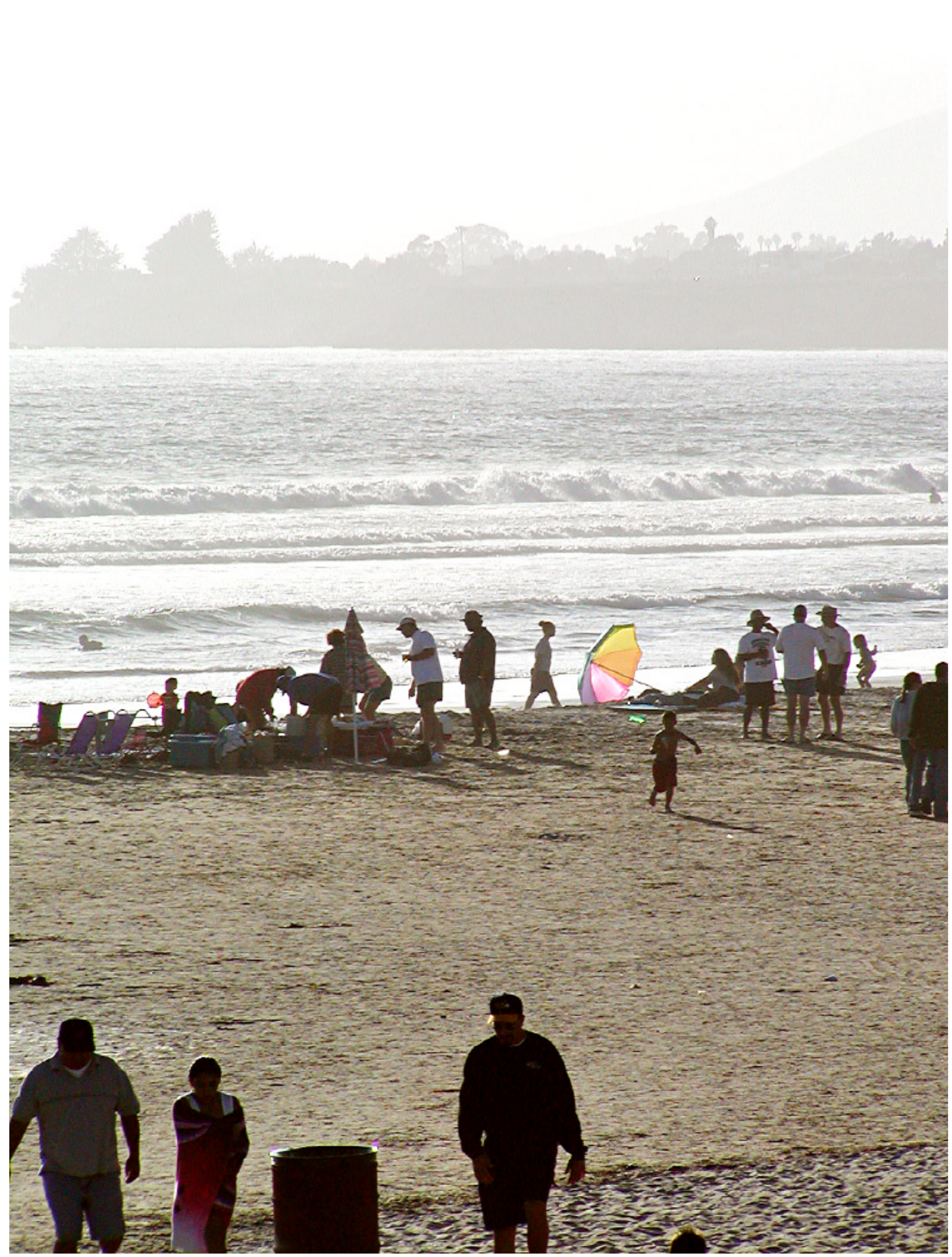
V. Write it up and publish it in theory conferences and journals

VI. Implement and communicate your results with the community your problem came from, discover related problems, repeat

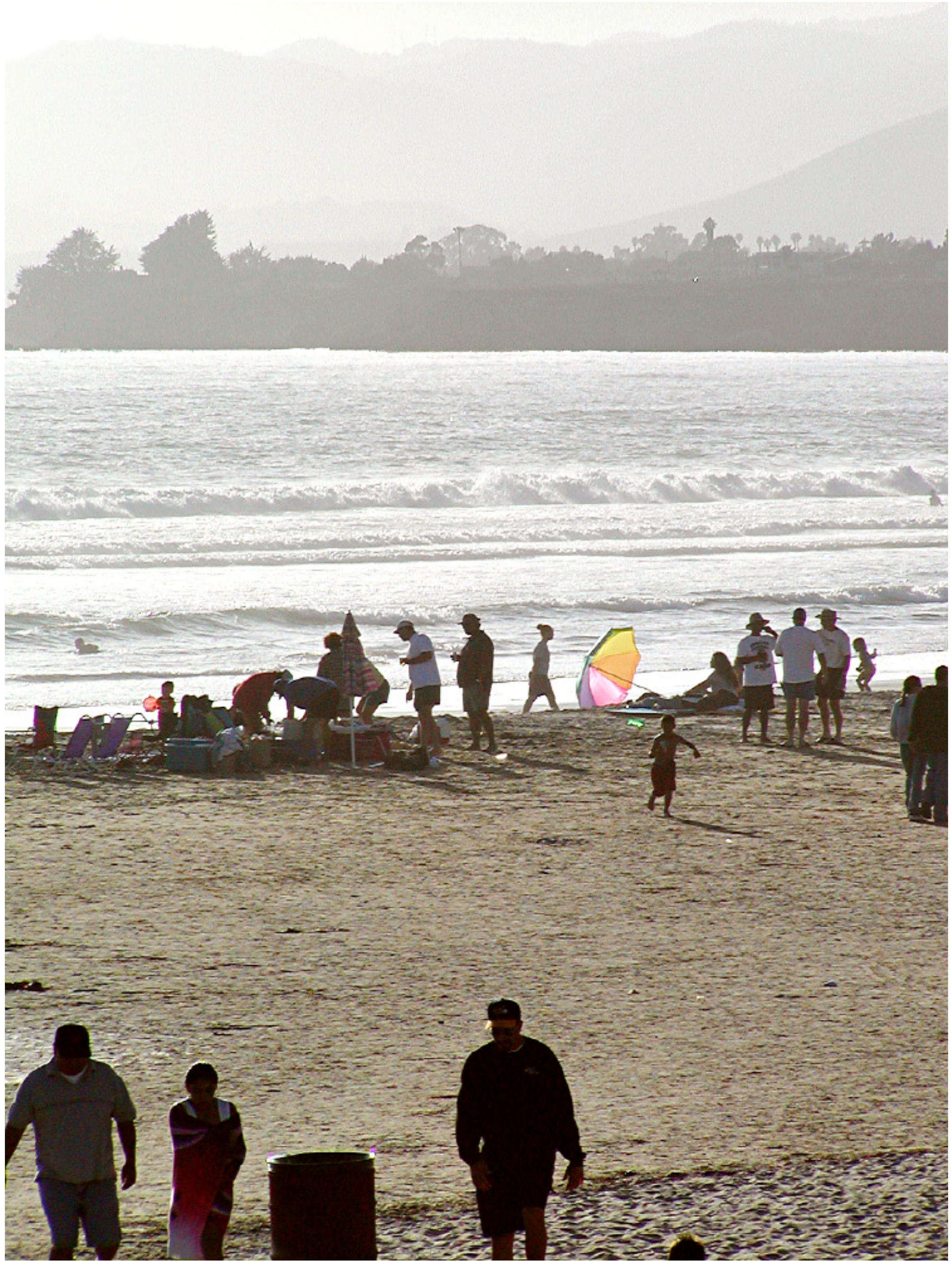
Application: photograph enhancement

e.g. lighten foreground of this picture without losing background detail

Top enhanced, bottom too dark



Bottom enhanced, top too light



Combination of top and bottom enhancements

Practical problem:

How to automatically find good split of picture into pieces to be enhanced separately?

Separation should have high contrast
to make different treatment of pieces less apparent

Pieces should be large
don′t want separate pieces for each little shadow

Theoretical abstraction:

Given planar graph with weights on each vertex, and given a target weight W

Can the graph be split into two connected subgraphs each with total weight at least W?

Graph vertices represent contiguous blocks of pixels connected by low-contrast adjacencies, weight is number of pixels

W represents requirement that pieces be large, could be set by user

Status:

No algorithm known, couldn′t find prior work
Might be NP-hard but approximating target weight would be ok

Application:

Building wiring design

Problem:

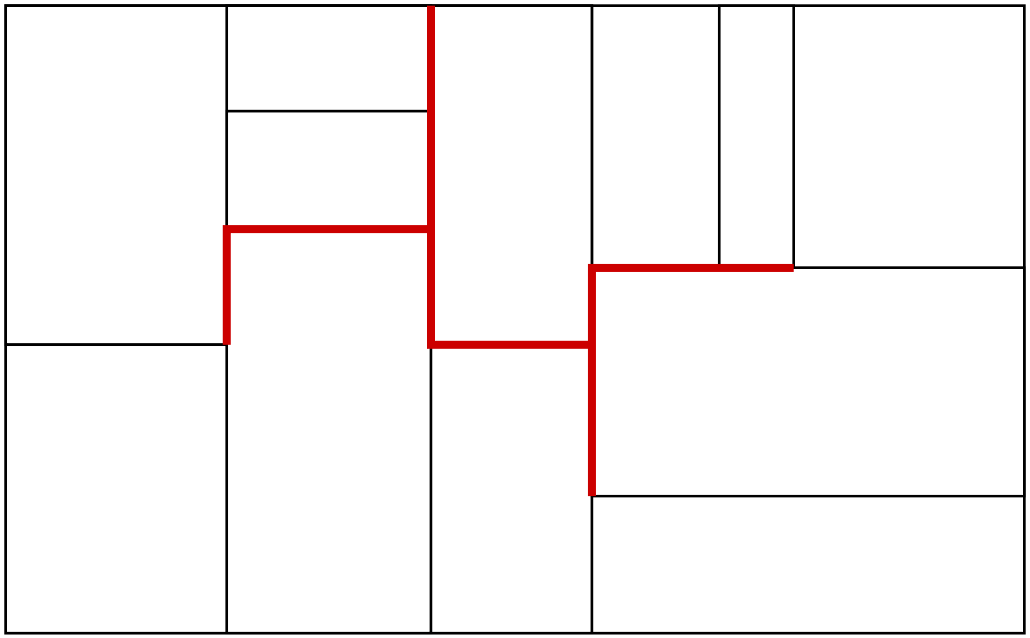
(from Naoki Katoh, Japanese architecture professor)

Given a rectangle partitioned into smaller rectangles

Find a tree touching each rectangle (including outer one)

Tree edges must lie along rectangle sides

Minimize total length



Status:

No algorithm known
(maybe not even a guaranteed approximation ratio)

Seems likely to be NP-complete but no proof known

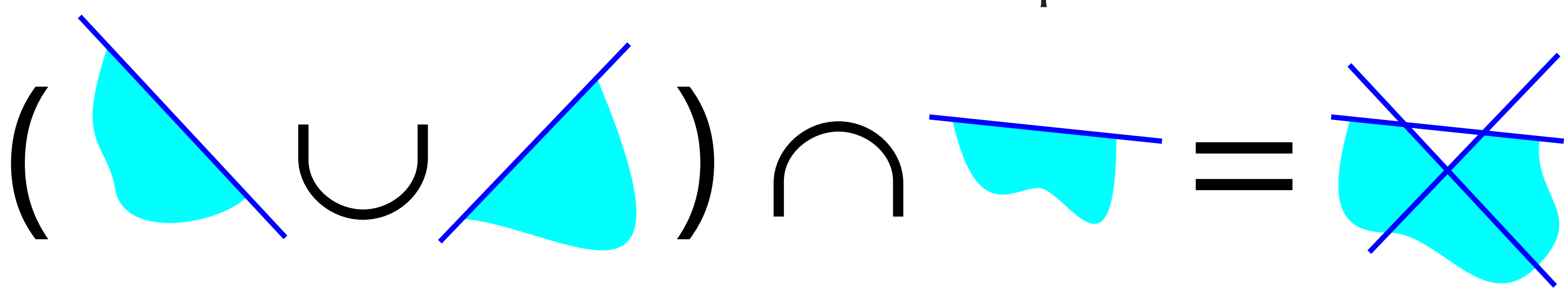
Application:

Communication network design

find spanning tree in a graph, optimizing nonlinear combination of two quantities e.g. cost and failure probability

Problem:

Given a formula representing a shape as unions and intersections of lower halfplanes



If formula has n terms, how many sides can final shape have?
Accurate bounds would improve network design algorithm analysis

Status:

Best lower bound: sides ≥ const · n alpha(n)
Best upper bound: sides ≤ const · $n^{4/3}$

Application:

Robust statistics

fit two-dimensional plane to higher-dimensional data points
insensitive to large number of arbitrary outliers

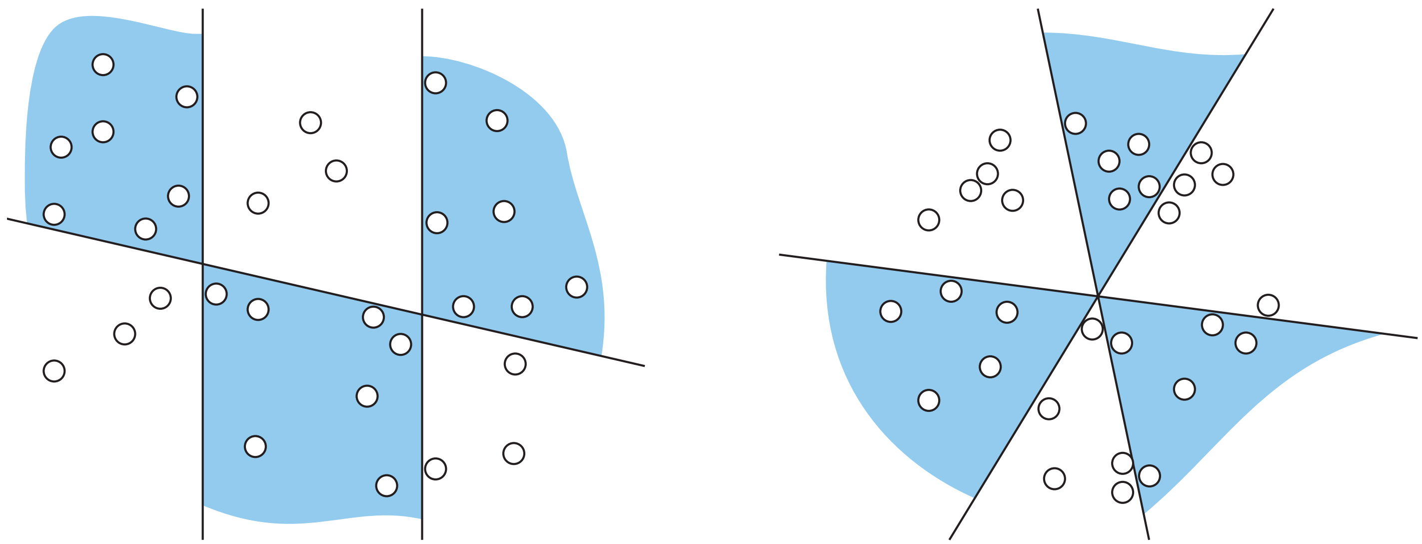
Problem:

Given a set of n points in two-dimensional plane

Find three subsets s.t. any line passes outside at least one subset
Want each subset to contain as many points as possible

Status:

Can always guarantee each subset contains at least n/6 points



There exist point sets where some subset must have ≤ n/4.622 points

Can we narrow this gap?

Application:

Exact graph coloring algorithms

Applications of application:

Parallel scheduling
Compiler register allocation
etc…

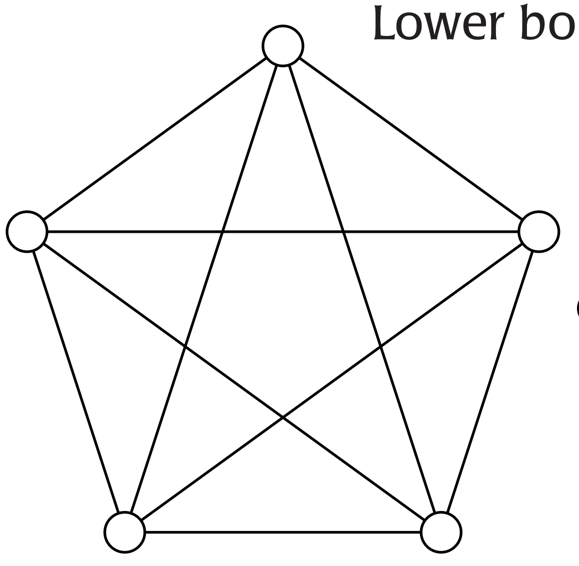
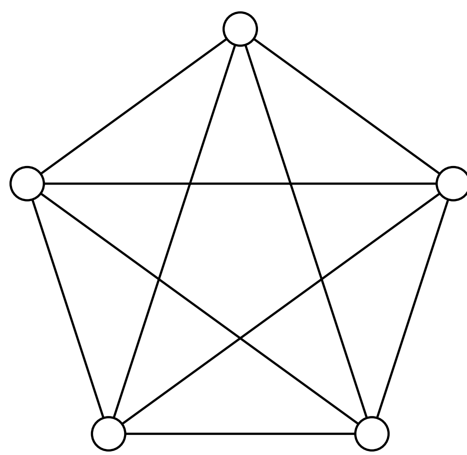
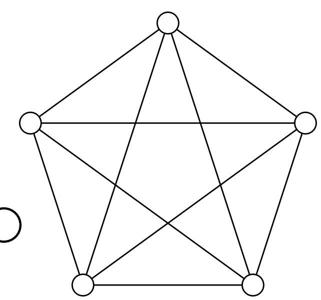
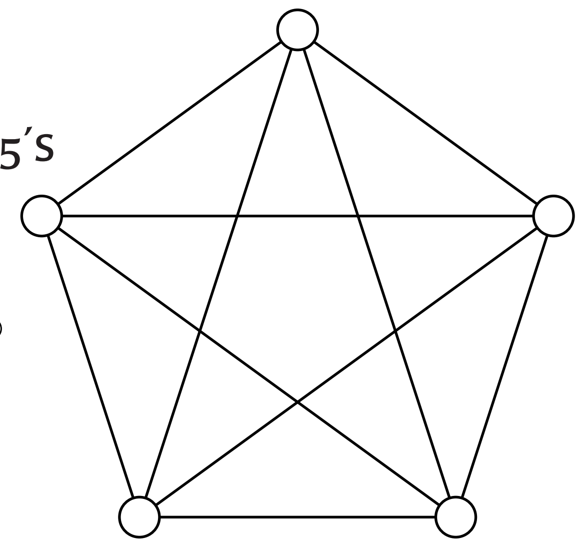
Problem:

How many maximal bipartite induced subgraphs
can an *n*-vertex graph have?

Status:

No nontrivial upper bound

Lower bound $10^{n/5}$ from disjoint union of K_5's

Application:

Provide theoretical justification for apparent difficulty of protein crystallography problems:
given known protein sequence, 3d electron density map,
find 3d positions of protein atoms

Problem:

Given: n by n matrix of 0's and 1's, sequence of n^2 0's and 1's

0 1 1 0 1
1 0 1 0 0
1 1 0 1 0
0 1 1 1 1
1 0 1 1 0

0 1 0 1 1 1 0 1 1 0 1 0 0 1 0 1 1 1 1 0 1 0 1 1 0

Can we find a path through adjacent matrix entries,
covering each matrix entry exactly once,
with values matching the given sequence?

Status:

Should be NP-complete
No proof known

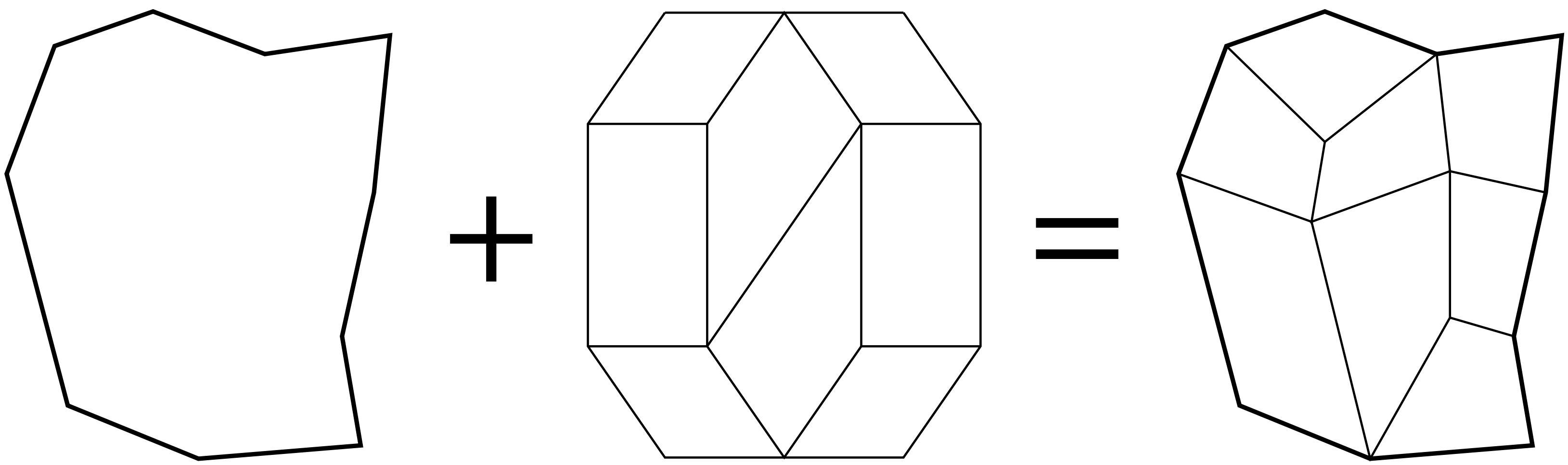
Application:

Finite element simulation

Problem:

Given: polygonal domain, mesh topology
(planar graph, all faces triangles or squares,
outer vertices matched up with domain vertices)

Can mesh be drawn in the plane with all faces convex?



Status:

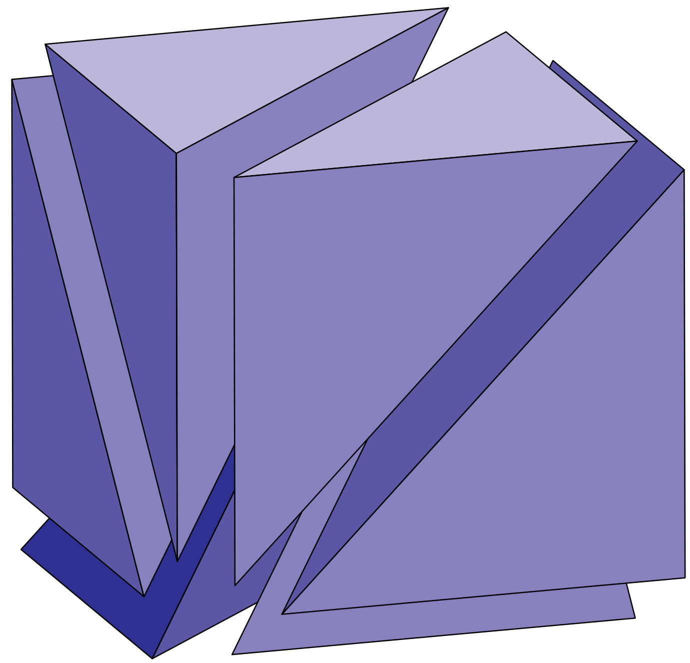
No efficient algorithm known
Not known to be NP-hard
Some practical success with ad-hoc heuristics

Application:

Finite element simulation

Problem:

It's known that the cube can be divided into tetrahedra s.t. all angles between adjacent faces are non-obtuse



But can we do it s.t. all angles are acute?

Status:

All-acute tetrahedralization of infinite slab (region between two parallel planes) is known [E. & Ungor]

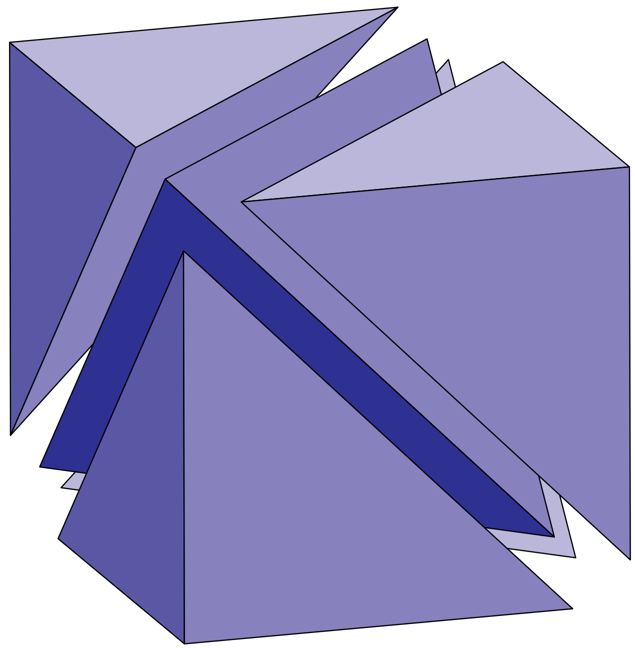
If possible, cube requires a large number of tetrahedra [Erickson]

Application:

None (purely of mathematical interest)

Problem:

Previous cube tetrahedralization used six tetrahedra,
but an improvement to five tetrahedra is possible and optimal:



How many higher dimensional simplices are needed
to triangulate a higher dimensional hypercube?

Status:

Best upper bound is slightly smaller than n!
Best lower bound is slightly larger than sqrt(n!)

Application:

Maybe someone can come up with one?

Problem:

Given n “centers” in the plane
Find n disjoint circles centered at those points
Maximizing total area

Status:

Seems likely to be NP-complete, but I have no proof
Polynomial-time approximation scheme exists

Replacing area by perimeter allows polynomial solution