Discrete Mathematics for Computer Science

Lecture 3

Sets; Set Operations

Introduction

Theoretical construction of mathematics	Practice
Logic → Sets → Everything is a set!	Logic — *Necessary to understand mathematical statements*
1- Natural numbers 2- Integers 3- Rationals 4- Reals numbers 5- Vector spaces……	Sets — *Sets are intensively used in computer science!* *(Theoretical computer science: sets are everywhere* *Practical computer science: Algorithms work on sets of data, e.g. Lists, Trees, Graphs, …)*

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Outline

- Basic definitions for sets
- Operations to build sets

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Sets

- Collection of objects (not ordered)
- Denoted using braces
  - {1,2,a,b}
  - {} usually denoted by ∅
- Sets can contain sets!
  - {{1},{2},{a},{b}}
  - {∅}
- Two sets are equal if they contain the same elements
  - {1,2,a,b} = {a,1,2,b} ≠ {{1},{2},{a},{b}}
  - ∅ ≠ {∅} ≠ {{∅}}

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Sets of numbers

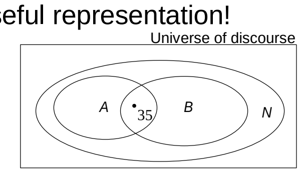
- Natural numbers
  - N={0,1,2,3,…}
- Integers
  - Z={…,-3,-2,-1,0,1,2,3,…}
- Rational numbers
  - Q is the set of fractions of integers like $\frac{-2}{3}$
- Real numbers
  - R is the set of rational numbers together with some non rational numbers like $\sqrt{2}, \pi, e, \ldots$

Appurtenance and inclusion

- Appurtenance
  - 1 is an *element* (a *member*) of N
  - N *contains* 1
  - Denoted by 1∈{1,2,a,b}
- Inclusion
  - A is a *subset* of B : ∀x (x∈A → x∈B)
  - Denoted by A ⊆ B or B ⊇ A
  - Note that for any sets A and B
    - ∅⊆A (because (F→T) = T)
    - A⊆A
    - A ⊆ B and B ⊆ A implies A=B
- Note that 1∈N while {1}⊆N

Venn Diagrams

- Very useful representation!



- A= natural numbers divisible by 5
- 35∈A
- B= natural numbers divisible by 7
  35∈B
- A⊆N and B⊆N

Cardinality

- Number of elements in a set
- Denoted by |A|
- Examples:
  - |{1,2,a,b}|=4
  - |∅|=0 , |{∅}|=1 , |{{∅}}|=1
  - |N|=infinity
- Finite set: cardinality is a natural number

Outline

- Basic definitions for sets
- How to build sets?

Enumeration

- Simplest way:
  - N={1,2,3,…}
  - E={1,2,3,…,9,10}
- Problems:
  - Often not rigorous
  - Can be very heavy to write
- Example: the set of numbers that are divisible by 5 and 7 and 11?
  - F={5,7,11,35,55,77,385,…}
  - IQ test!

Set builder

- Use a propositional function to describe the elements
- { x | P(x) } reads "the set of x (in the universe of discourse) such that P(x)"
- Example: the set of numbers that are divisible by 5 and 7 and 11?
  - E = {$x \in N$ | x is divisible by 5 AND x is divisible by 7 AND x is divisible by 11 }
  - E = {1,3,5,7,…}={$x \in N$ | x is odd}

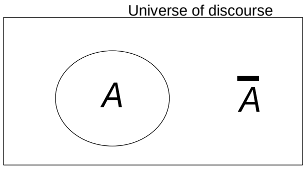
Specify the universe of discourse

Complement

- Complement of A: elements of the universe which not in A

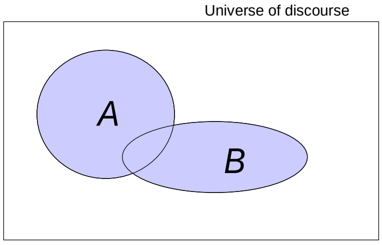
$$\bar{A} = \{x \in U \mid x \notin A\}$$

Universe of discourse



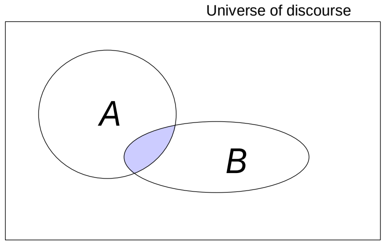
Union

$x\in A\cup B \leftrightarrow x\in A \lor x\in B$



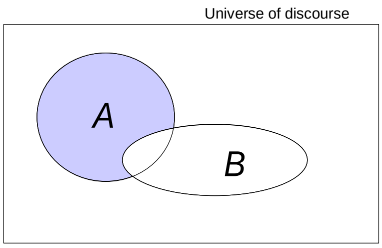
Intersection

$A\cap B = \{x\in A \mid x\in B\}$



Difference

$A-B = \{x\in A \mid \neg\, x\in B\}$



Properties of $\cup$, $\cap$ and -

- Table 1 p 89

à Compare to Table 5 page 24

- Example: De Morgan's law

$$A\cup B = \{x\in U \mid x\in A \lor x\in B\}$$

$$\overline{A\cup B} = \{x\in U \mid \neg(x\in A \lor x\in B)\}$$

De Morgan's law $$= \{x\in U \mid \neg(x\in A) \land \neg(x\in B)\}$$

$$= \{x\in U \mid x\in \overline{A} \land x\in \overline{B}\}$$

$$= \overline{A}\cap\overline{B}$$

Power set

- The power set of A is the set of all subsets of A
- Denoted by P(A)

$$x \in P(A) \leftrightarrow x \subseteq A$$

- P({1,2,a}) ?
  - Subset of cardinality 3: {1,2,a}
  - Subset of cardinality 2: {1,2} , {1,a} , {2,a}
  - Subset of cardinality 1: {1} , {2} , {a}
  - Subset of cardinality 0: ∅
- Remember: $A \in P(A)$ and $\emptyset \in P(A)$

Playing cards

- Let E be a deck of cards
- Poker: give 5 cards to each players
- What are the possible hands?
- 1 hand is a subset of E with 5 elements

$$H=\{ x \in P(E) \mid |x| = 5 \}$$

- Nice, isn't it?

Cartesian product

- Ordered pair: (a,b)
  Note that $(a,b) \neq (b,a)$!
- Ordered 3-tuple: (a,b,c)
- Ordered n-tuple: $(a_1,a_2,\ldots,a_n)$
- Cartesian Product:

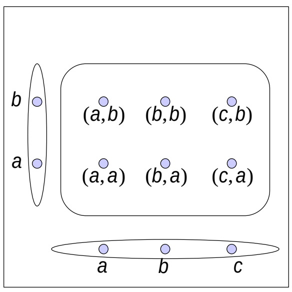
$$A \times B = \{ (x,y) \mid x \in A \text{ AND } x \in B \}$$

$$A \times B \times C = \{ (x,y,z) \mid x \in A \text{ AND } x \in B \text{ AND}\}$$

$$A_1 x A_2 x \ldots x A_n = \{ (x_1,x_2,\ldots,x_n) \mid x_1 \in A_1 \text{ AND } x_n \in A_n \text{ AND } \ldots \text{ AND } x_n \in A_n \}$$

Cartesian product: Example 1

- A={a,b,c} and B={a,b}
- AxB contains
  - (a,a) and (a,b)
  - (b,a) and (b,b)
  - (c,a) and (c,b)

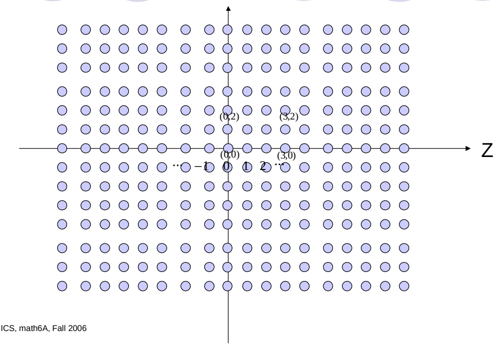


Cartesian product: Example 2

- ZxZ={ ………

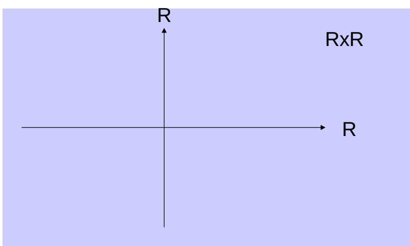
…,(-2,-2),(-2,-1),(-2,0),(-2,1),(-2,2),…
…,(-1,-2),(-1,-1),(-1,0),(-1,1),(-1,2),…
…,(0,-2),(0,-1),(0,0),(0,1),(0,2),…
…,(1,-2),(1,-1),(1,0),(1,1),(1,2),…
…,(2,-2),(2,-1),(2,0),(2,1),(2,2),…
………}

Cartesian product: Example 2



Cartesian product: Example 2

- RxR={ (x,y) | x∈R ∧ y∈R }



Extra

- Exercises 7 and 13 and 15
- Exercises 14d, 19 (also draw Venn Diag.)