Joint Transmission Scheduling and Routing for Low-Power Wireless Sensor Networks is Polynomially Solvable

Yannis Paschalidis
yannisp@bu.edu
http://ionia.bu.edu/

CENTER FOR INFORMATION AND SYSTEMS ENGINEERING (CISE)
AND DEPARTMENT OF MANUFACTURING ENGINEERING
BOSTON UNIVERSITY

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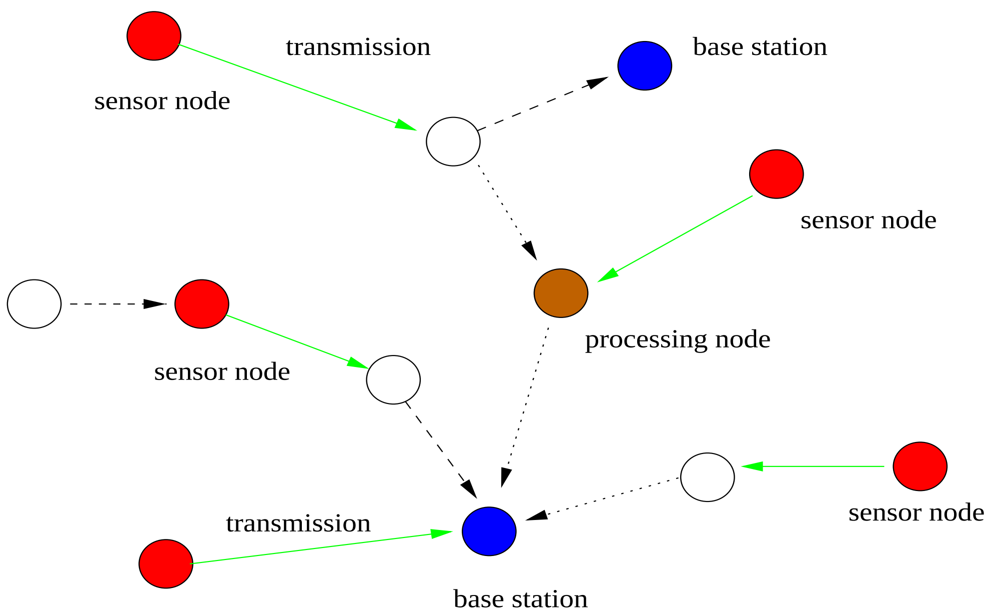
Wireless Sensor Networks (SNETs)



- Nodes communicate **wirelessly**, generally with **low power levels**.
- Nodes can only do one task at a time (send or receive) and concurrent transmissions cause **interference**.
- Stringent **energy constraints** (i.e., random MAC is often inefficient, benefits from multihop).
- Optimization is a **necessity**.

Objective and Approach

- Formulation:

Maximize	Utility of transmission rates
subject to	Physical layer constraints
	Node exclusion constraints
	Fairness Constraints

- Key ideas: **characterize the region of achievable rates** and optimize efficiently.
- Our approach is based on **decomposition** methods in large-scale optimization.
- Can we trade-off energy (i.e., network lifetime) vs. performance ?
- Can we address scalability issues ?
- Can we recover from node failures ?

OUTLINE

- The SNET model
- The achievable region and structural properties
- A large-scale decomposition approach
- Complexity analysis
- Energy conservation
- Numerical results
- Concluding Remarks

SNET Model

- N **sensors**, M **gateways** (sinks), $\bar{p}_i$ max power of node i.
- **Traffic class**: (information content, utility, OD pair). K traffic classes. $s(k), d(k)$: source, destination for class k.
- (i, j, k)-transmission: class k traffic from node i to node j.
- **Power Limits**: $p_{ijk} \leq \bar{p}_i$.
- SINR for an (i, j, k)-transmission:

$$\gamma_{ijk} = \frac{p_{ijk} G_{ij}}{\eta W + \sum_{v=1}^{K} \sum_{l=1, l \neq i}^{N+M} \sum_{u=1}^{N+M} p_{luv} G_{lj}}$$

- **Node exclusion** constraints:
  - No multicast; transmissions are from one node to another.
  - Sensors cannot transmit and receive simultaneously.
  - Sensors cannot receive from multiple sensors simultaneously.
  - (WLOG) gateways can receive from multiple sensors.

Valid Transmissions and Achievable Rates

- Net flow rate for an (i, j, k) or a (j, i, k) transmission:

$$r_{ijk} = W \log(1 + \gamma_{ijk}) - W \log(1 + \gamma_{jik}).$$

- **Linear (low power) approximation**:

$$r_{ijk} = (p_{ijk} G_{ij} - p_{jik} G_{ji})/(\eta \ln 2) \quad \text{or} \quad \mathbf{r} = \mathbf{H}\mathbf{p}.$$

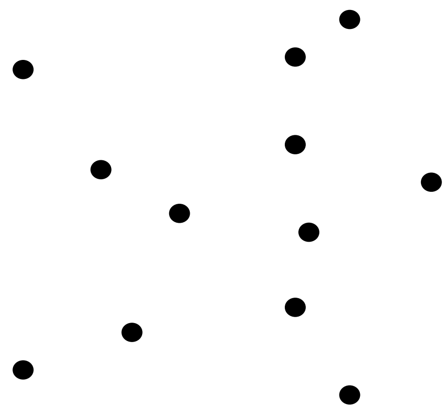
- Set of **valid transmission schemes**

$$\mathscr{P} \stackrel{\triangle}{=} \{\mathbf{p} \mid \text{power limits \& node exclusion constraints}\}$$

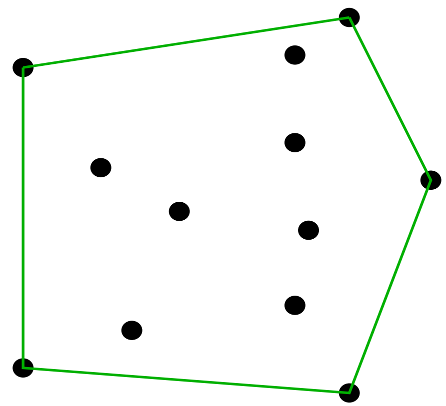
- Region of **achievable transmission rates**:

$$\mathscr{R} = \{\mathbf{r} \mid \mathbf{r} = \mathbf{H}\mathbf{p},\ \mathbf{p} \in \mathscr{P}\}.$$

Achievable Region (cont.)

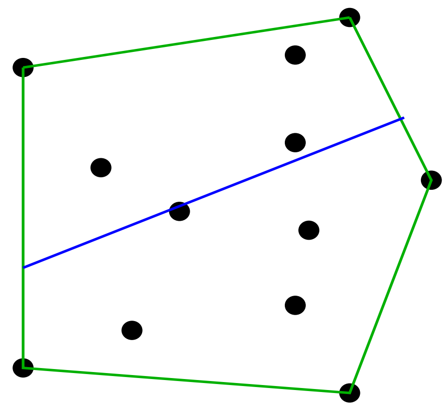


Achievable Region (cont.)



- Enlarge the achievable region by **time-sharing**. $\text{Conv}(\mathscr{P})$ and $\text{Conv}(\mathscr{R})$ are polytopes.

Achievable Region (cont.)



- Enlarge the achievable region by **time-sharing**. Conv($\mathscr{P}$) and Conv($\mathscr{R}$) are polytopes.

- Introduce **fairness** and **flow-conservation** constraints.

$$\mathscr{S} \triangleq \left\{ \mathbf{r} \mid \mathbf{A}\mathbf{r} \leq \mathbf{b}, \sum_{j=1}^{N+M} r_{ijk} = 0, \forall i \neq s(k), d(k), \forall k \right\}$$

- **Utility Maximization Problem**:

$$\begin{aligned} \max \quad & F(\mathbf{r}) \\ \text{s.t.} \quad & \mathbf{r} \in \text{Conv}(\mathscr{R}) \cap \mathscr{S}. \end{aligned}$$

Earlier Work

- Existing work has concentrated on ad-hoc wireless networks.
- [Gupta & Kumar]: (information theoretic) capacity of wireless networks. [Kumar et al.]: improved random MAC protocols.
- [Johansson, Xiao, Boyd], [Lin & Shroff]: joint routing & power control for high SINR.
- [Radunovic & Le Boudec]: heuristic policies for UWB networks.
- [Toumpis & Goldsmith]: enumeration approach ... but Conv($\mathscr{R}$) has **MANY** extreme points.

Our approach takes advantage of **time-sharing**, considers **general utility functions**, incorporates **fairness** considerations, and can **handle large networks**.

Complexity

- Formulation ($\mathbf{r}^n$ are extreme points of Conv($\mathscr{R}$)):

$$
\begin{array}{lll}
\min & -F(\mathbf{r}) & \\
\text{s.t.} & \mathbf{r} - \sum_{n=1}^{L} \alpha_n \mathbf{r}^n = 0, & (\boldsymbol{\lambda}) \\
& \sum_{n=1}^{L} \alpha_n = 1, & (\mu) \\
& \sum_{j=1}^{N+M} r_{ijk} = 0, \ \forall i \neq s(k), d(k), \forall k, & (\boldsymbol{\nu}) \\
& \mathbf{A}\mathbf{r} \leq \mathbf{b}, & (\boldsymbol{\sigma}) \\
& \alpha_n \geq 0, \ n = 1, \ldots, L. &
\end{array}
$$

- **Optimal Policy**: TDMA, use **transmission mode** $\mathbf{r}^j$ w.p. α_j.
- Assuming concave utility this is a convex optimization problem. Note that the formulation has **exponentially many** decision variables.
- **Theorem** (**Polynomial Solvability**) *Under some regularity assumptions the problem can, in principle, be solved in polynomial time.*

  Not practical as proof relies on the ellipsoid method.

A decomposition approach

- Developed a **cutting-plane** method for the dual problem.
- **Dual problem**: Feasibility constraints include

$$\mu - \boldsymbol{\lambda}'\mathbf{r}^n \geq 0, \quad n = 1, \ldots, L.$$

- **Restricted master problem**: $\mathbf{r} \in \text{Conv}(\{\mathbf{r}^1, \ldots, \mathbf{r}^m\})$, $m \leq L$.
- **Restricted dual problem**: Feasibility constraints include

$$\mu - \boldsymbol{\lambda}'\mathbf{r}^n \geq 0, \quad n = 1, \ldots, m.$$

- m-th iteration: Given an optimal **primal**-**dual** pair $(\mathbf{r}^{(m)}, \boldsymbol{\alpha}^{(m)}; \boldsymbol{\lambda}^{(m)}, \mu^{(m)}, \boldsymbol{\nu}^{(m)}, \boldsymbol{\sigma}^{(m)})$ find **extreme point** $\mathbf{r}^{m+1} \in \text{Conv}(\mathscr{R})$ s.t. $\mu^{(m)} - \boldsymbol{\lambda}^{(m)'}\mathbf{r}^{m+1} < 0$.

The Separation Problem (Subproblem)

$$\begin{aligned} \max \quad & \boldsymbol{\lambda}'\mathbf{r} \\ \text{s.t.} \quad & \mathbf{r} \in \text{Conv}(\mathscr{R}). \end{aligned}$$

- Reduces to an integer linear programming (ILP).
- Optimal solutions $\mathbf{r}^*$ correspond to a valid **transmission mode** $\mathbf{p}^*$: determines which sensors transmit.
- **Theorem** *Transmitting sensors transmit at max power.*
- **Theorem** *Subproblem is equivalent to a* **maximum weighted matching problem***.*
- Complexity: $O(KN(N+M)+(M+1)^3N^3)$.

Refinements and Extensions

- **Removing the linear approximation of transmission rates**: Once we obtain the structure of the policy we can remove the (low power) linear approximation and obtain a new (refined) policy.
- Can effectively deal with **node failures** (re-optimize).

Trading-off energy consumption vs. utility

- View optimal value of the utility maximization problem, $F(\bar{\mathbf{p}})$, as function of power limit vector $\bar{\mathbf{p}} = (\bar{p}_1, \ldots, \bar{p}_N)$.
- **Energy minimization**:

$$\begin{aligned} \min \quad & \phi \\ \text{s.t.} \quad & \bar{F}(\phi \bar{\mathbf{p}}_0) \geq F_{\min}. \end{aligned}$$

- Let T the SNET **lifetime** (time until first node battery depletion). Let $\hat{F}(T)$ the max utility we can achieve subject to lifetime $\geq T$. Can efficiently solve (convex in $1/T$)

$$\max_T (\hat{F}(T) - \zeta/T).$$

  Other variants:

  - maximize SNET lifetime s.t. minimum utility constraint;
  - maximize utility s.t. min lifetime constraint.

Numerical Results

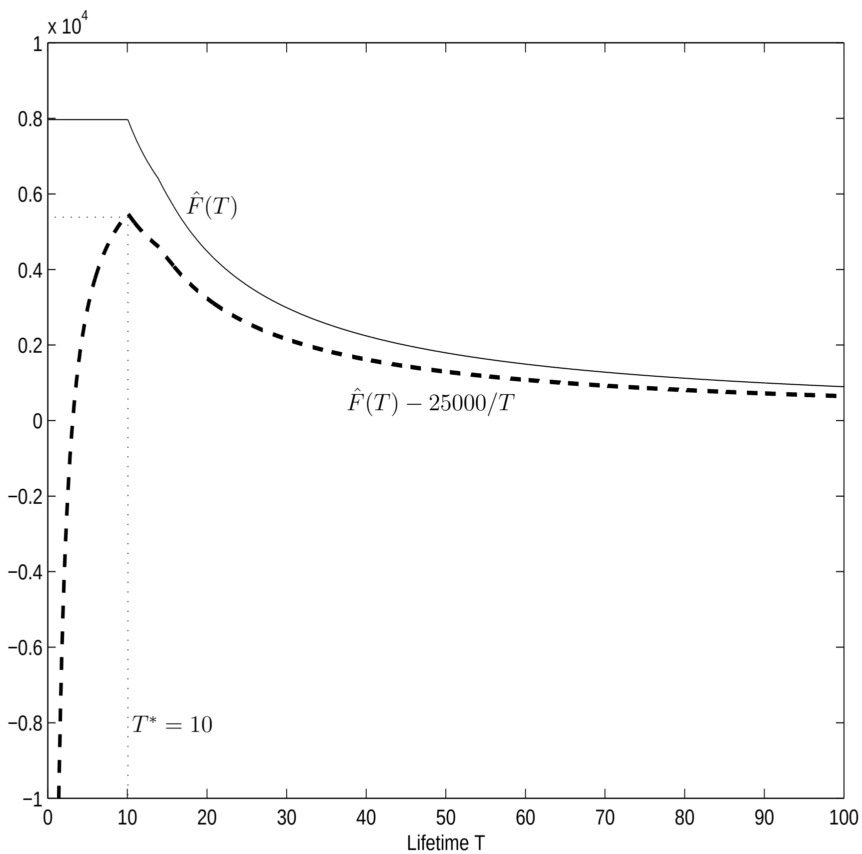
- **Accuracy of our approach and efficiency.** Considered SNET with single gateway; max. total throughput s.t. fairness constraints (power levels at 0.1 Watts)

N	Enumeration	Time	Decomposition	Time	Single-hop
2	14.44	0.02	14.44	0.01	14.4
3	122.28	0.02	122.28	0.01	122.2
4	689.16	0.13	689.16	0.02	167.6
5	7962.63	63.4	7960.87	0.02	582.3
6	out of memory	-	6339.97	0.03	191.9

- We can solve problems with 50 nodes in less than 1 min !
- Multihop better than singlehop (by 3000% !).
- Our approach is accurate at much higher power levels (e.g., 40db).

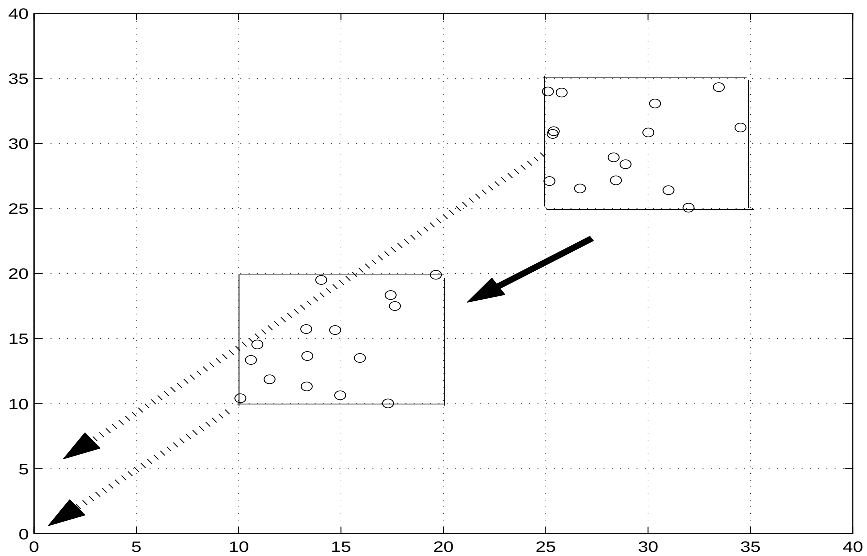
Numerical Results (cont.)

Utility vs. lifetime (5-node case):



Numerical Results (cont.)

Another example

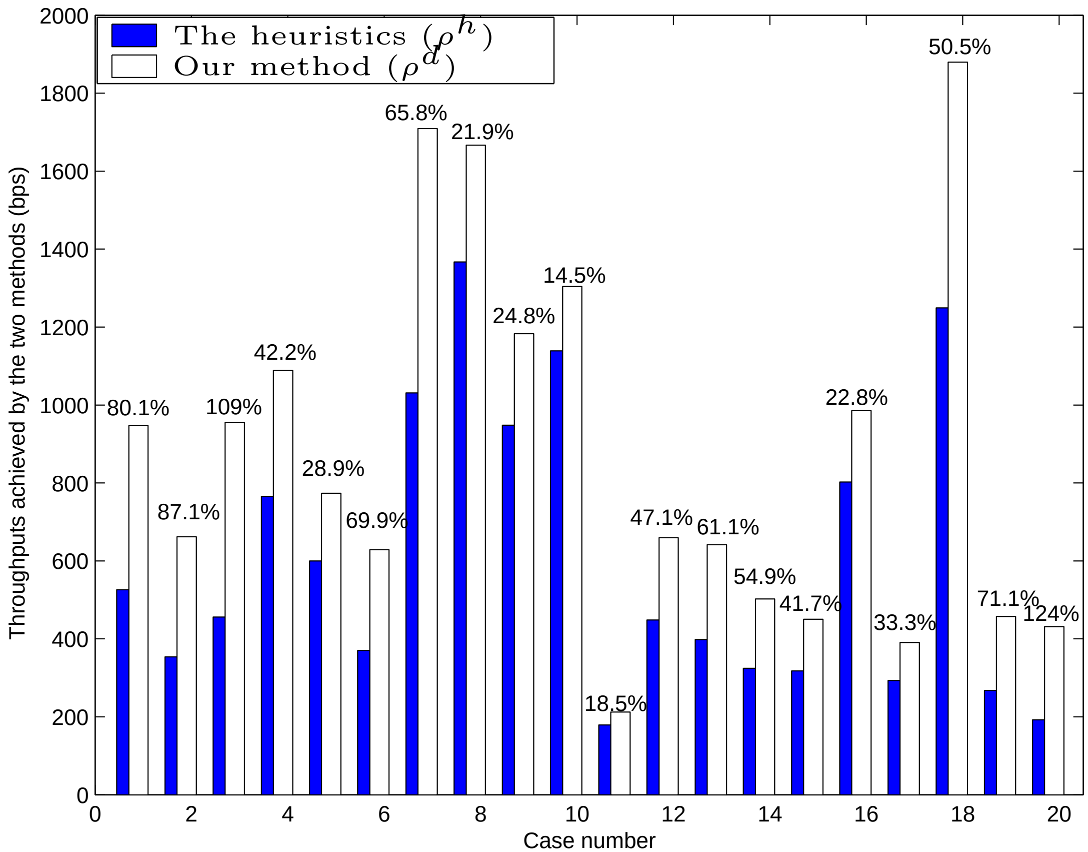


N	Decomposition	Single-hop	2-hop
20	672.99	26.59	625.07
26	851.20	27.09	633.41
30	949.77	31.26	691.12

Last row: 37.5% improvement over 2-hop policy.

Comparisons with recent heuristics

We considered recently proposed heuristics by [Radunovic & Le Boudec], a 40-node SNET, powers at 1 mWatt. Randomly generated SNETs and obtained



Summary and Final Thoughts

- Considered a framework for **transmission scheduling and routing** that
  - accommodates sensor diversity (utility maximization) and
  - imposes explicit fairness constraints.
- Developed an **efficient decomposition approach** for solving the problem that
  - produces TDMA policies which time-share among several transmission schemes;
  - can handle node-failures;
  - can trade-off utility vs. energy; and
  - can solve large problems.
- Established **polynomial solvability** of the utility optimization problem.