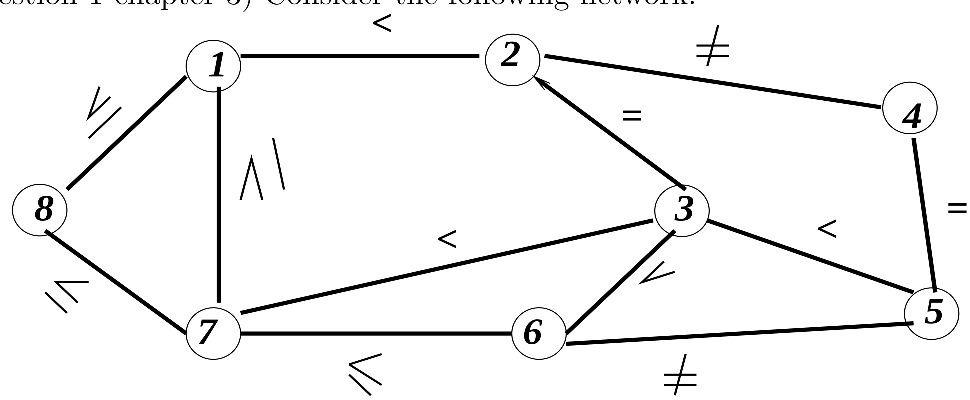
Rina Dechter, Spring-2009

ICS 275, Assignment 2

Read chapters 3 in book and answer the following questions. Whenever you are asked to generate arc, path consistent network you should not only show the end-result but show also how it is derived. You can either implement one of the known algorithms hand-in your code, or hand-simulate one of the known algorithm. Stop when your simulation takes more then one page.

1. (Question 1 chapter 3) Consider the following network.

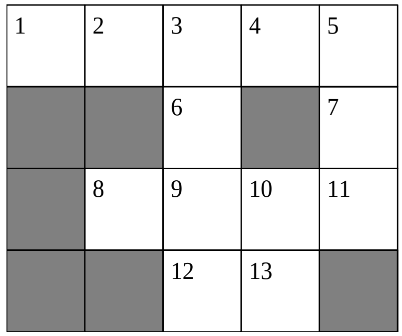


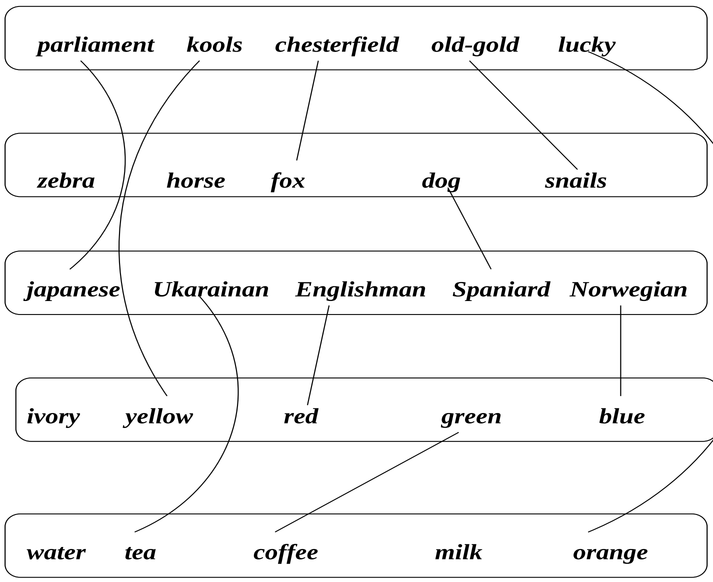
   Assume that each variable has a domain of $\{1, 2, 3, 4\}$.

   a. Find an equivalent arc and path-consistent network. Is the path-consistent network minimal? Is it backtrack-free?
   b. Apply distributed arc-consistency to the dual graph. Show the computations relevant to two adjacent constraints of your choice, only. For these two constraints show the first round of outgoing and incoming messages and the subsequent local update.

   c. Analyze the complexity of distributed arc-consistency.

2. (Question 2 chapter 3) Consider the CSP formulation of the Zebra problem where you have 25 variables, divided into clusters and where the domains are the houses numbers (see attached figure). Is the problem arc-consistent? If not specify an equivalent arc-consistent problem. Is the problem path-consistent?





3. (**not to be turned-in**)(question 5 chapter 3) Consider the crossword puzzle described in chapter 1 formulated in its dual representation (namely full words are variables and the constraints are binary). The possible words for this puzzle are:

$$\{HOSES, LASER, SHEET, SNAIL, STEER, HIKE, ARON, KEET, EARN, SAME$$

$$RUN, SUN, LET, YES, EAT, TEN, NO, BE, US, IT\}$$

   (a) Is the problem arc-consistent? If not, find an equivalent arc-consistent network. Is the network path-consistent?.

4. (question 6 chapter 3) Prove that the minimal network is always path-consistent: Any consistent pair of values can always be extended to any third variable.

5. (question 7 chapter 3, extra credit) Prove that a bi-valued non-empty path-consistent network is consistent. Prove that it is also the minimal network.

6. Consider 2-cnf formulas (conjunction of clauses of length 2).

   (a) Describe an algorithm for enforcing arc-consistency of 2-cnf formulas using resolution.

(b) Describe an algorithm for enforcing path-consistency on 2-cnf formulas, using resolution.

7. (question 10 and 11 chapter 3)

   (a) Let $\mathcal{R}$ be an arbitrary 3-graph coloring problem. Discuss what would be the effect of enforcing 2-consistency on $\mathcal{R}$, what would be the effect of enforcing 3-consistency, or 4-consistency on $\mathcal{R}$.

   (b) In general, what would be the effect $2, 3, 4, ...k-1, k, k+1$-consistency on a k-coloring problem?

8. (question 17, chapter 3) Generate an arc and path-consistent network which is equivalent to the network:

$$D_x: \ x \in [3, 10], \quad D_y: \ y \in [5, 12], \quad R_{xy}: \ x + y = 10$$

$$D_z: z \in [-11, 12], \quad R_{yz}: y + z \leq 3$$

9. (question 19, chapter 3) Derive the constraint alldifferent for the speaker problem presented in Section 3.5.2 and make this global constraint arc-consistent