

## ICS 275, Assignment 7

This homework is based on Chapters 9, 10 and 13. It will count twice compared with regular homeworks.

1. (question 2 chapter 9 and question 3 chapter 10) Consider the crossword puzzle:

1		2		3
	4		5	
6		7		
8				

### Word List

aft	laser
ale	lee
eel	line
heel	sails
hike	sheet
hoses	steer
keel	tie
knot	

- (a) Is the crossword puzzle an acyclic constraint network? Justify your answer.
- (b) Consider the (primal) constraint graph of the problem and an ordering that has a minimum induced-width.
  - a. Create the set of cliques that will be generated by tree-clustering and provide one tree-decomposition.
  - b. Using the tree of cliques you created generate the actual constraint for each cliques so that the original problem is equivalent to the tree-structured problem. State explicitly the join-tree you created.
  - c. Apply CTE (show the messages schematically).
  - d. Compare the performance of adaptive-consistency, JTC and CTE for this problem.
- (c) Suppose you want to solve the crossword puzzle using the cycle-cutset scheme. Discuss the approach. Can you bound the complexity of the cycle-cutset scheme for problems having this constraint graph? Compare with previous approaches.
- (d) Bound the complexity of solving problems having this constraint graph using the the non-separable-components method.

- (e) What are the best time-space guarantees you can get for this instance of the crossword puzzle. What are the best time/space guarantees you can provide for a crossword puzzle of size  $n \times n$ .
  - (f) In your opinion, which of the methods you have learned in the constraint class is best suited for cross-word puzzles?
2. (question 3 chapter 9) Consider the graph problem in Figure 9.7a of chapter 9 and assume that all the constraints are binary "not-equal" constraints and all the domains of the variables have the values  $\{1, 2\}$  except the domain of  $B$  is  $\{1, 2, 3\}$ . a. Generate a tree-decomposition. b. Apply CTE to this problem fully (show the messages exactly).
  3. (question 4, chapter 9) Consider a grid problem with 9 variables and binary constraints. Describe a join-tree decomposition created by JTC.
    - (a) What is the tree-width, hyper-width and separators of your decomposition.
    - (b) What is the time and space complexity of *CTE* on the decomposition.
    - (c) Show schematically how adaptive-tree-consistency will work on this problem. What would be the time and space complexity?
  4. Question 5 chapter 9
  5. (question 2 chapter 10) Prove that deciding the consistency of a constraint network having nonseparable components of size at most  $r$  can be solved in time exponential in  $r$  while in linear space.
  6. (question 5 chapter 10) Based on worst case complexity analysis, show an example of a problem class where the cycle-cutset scheme is superior to recursive conditioning and vice-versa. Show a class where recursive-conditioning is better than the cycle-cutset scheme.
  7. (question 2 chapter 13). The combinatorial auction problem was described in chapter 13 and in exercise 13 of chapter 5.
    - (a) Provide one way of formulating this problem as a constraint optimization problem. Demonstrate your formulation over a small problem (5 variables).
    - (b) Discuss the pros and cons of solving this problem by Branch and Bound algorithms vs bucket elimination.
  8. (question 4 chapter 13) consider a graph-coloring problem that has 4 variables  $(A, B, C, D)$ , where the domains of  $A$  and  $C$  are  $\{1, 2, 3\}$  and the domains of  $B$  and  $D$  are  $\{1, 2, \}$ . The constraints are not equal constraints between adjacent variables. Apply the elim-count algorithm to this problem. compare its performance with a simple search algorithm for counting.
  9. (extra credit) Question 6 chapter 13.