

COMPSCI 276
Homework Assignment 6
Fall 2007

Instructor: Rina Dechter

Due: Wednesday, Nov 28

Relevant reading: Pearl's chapter 5, Class Notes chapter 3, AND/OR paper.

1. (15) Question 5.1 in Pearl's book.
2. Consider the Bayes network DAG in Figure 1:

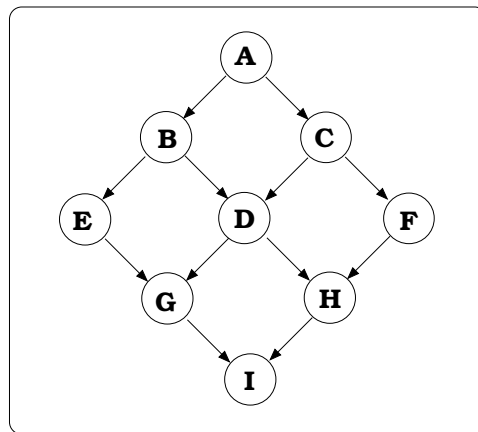


Figure 1: A Bayesian network

- (a) (10) Discuss two exact algorithms for finding the belief of $P(A|I = 0)$. Demonstrate their performance schematically and discuss their complexity.
- (b) (10) Assume you cannot record more than binary functions (defined on two variables), which exact algorithms can accommodate such restrictions? what is their time and space complexity?
- (c) (10) Apply the approximation algorithm mbe-bel($i=3$) for the task of finding the belief in A . Trace the algorithm's performance schematically (show functions, no numbers). What is the time and space complexity of the algorithm?
- (d) (10) Apply mbe-mpe($i=3$) to find an upper bound for the mpe of the network given $I = 0$. Trace the algorithms. Show how you construct an approximate mpe tuple.
- (e) (10) Propose a node duplication simplification for the network that will correspond to the mini-bucket scheme you used in your answer above. Discuss how a full bucket-elimination on your relaxed problem corresponds to the mini-bucket scheme.
- (f) (10) (extra credit) Run mini-bucket approximation, and IBP to compare the approximation quality on this network. (You can use REES for MBE-bel and IBP. This can be part of a project)

3. (15) Assume you are given a 4x4 directed grid (like in Figure 1 but having dimension 4 rather than 3).
 - (a) Provide an arc-labeled minimal dual graph for the 4x4 grid network.
 - (b) Generate a join-graph whose maximal cluster size is 4. Show the functions in each cluster.
 - (c) Show the schematic messages that would pass in one iteration of IJGP on your join graph.
4. This question investigates the AND/OR search space of the network in Figure 2, assuming each variable has 2 values in its domain. The CPTs are given in Figure 3. The CPTs for G , H and D are identical to the 3-dimensional CPT in Figure 3 and the CPTs for H and F are identical to the two-dimensional CPT in that figure.

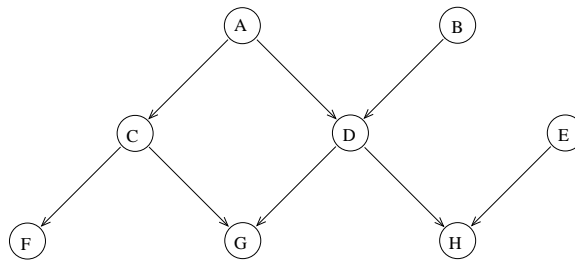


Figure 2: A directed graph

- (a) (5) Find and present a backbone pseudo tree of the network in Figure 2 whose depth is minimal (do the best you can). Call this tree T_1 .
 - (b) (10) Generate an AND/OR search tree driven by T_1 assuming each variable has at most two values.
 - (c) (5) Annotate the arcs with the appropriate weights.
 - (d) (10) What would be the computational cost of computing the probability of evidence $G = 0$ and $H = 1$ in such a network if you use depth-first search over the AND/OR search tree. Demonstrate the computation (compute the value of each node)
 - (e) (5) Can the AND/OR search tree be reduced to a smaller AND/OR search graph? if so, demonstrate.
 - (f) (5) Compare the time and space complexity of solving this problem by search with context-based caching vs bucket-elimination.
 - (g) (10) Assume that the CPT $P(x|y, z)$ is changed by making some entries deterministic, as follows: set the probability column of the first 2 lines to 1 and 0 in that order (from top to bottom). Do the same for the last 2 lines (e.g., we make $P(x = 0|y = 1, z = 1) = 0$). Show what would be the changes in the AND/OR search tree as a result.
5. (5) (Extra credit) Provide feedback on chapter 3 in the class notes: structure, typos, clarity. Does it help in your studies?

a	$p(a)$	b	$p(b)$	e	$p(e)$
0	0.3	0	0.6	0	0.7
1	0.7	1	0.4	1	0.3

y	x	$p(x y)$
0	0	0.10
0	1	0.90
1	0	0.30
1	1	0.70

z	y	x	$p(x y, z)$
0	0	0	0.25
0	0	1	0.75
0	1	0	0.60
0	1	1	0.40
1	0	0	0.10
1	0	1	0.90
1	1	0	0.20
1	1	1	0.80

Figure 3: Conditional probability tables