1. Consider the Bayes network DAG in Figure 1:

(a) (10) Discuss the performance of the bucket-elimination algorithm for finding the belief of $P(A|I = 0)$. Demonstrate its performance schematically (describe algebraically what function is computed in each bucket). What would be the complexity of the algorithm?

(b) (10) Apply the approximation algorithm mbe-bel(i=3) for the task of finding the belief in $A$. Trace the algorithm’s performance schematically (show functions, no numbers). What is the time and space complexity of the algorithm?

(c) (extra credit, 5) Apply the weighted mini-bucket algorithm wmbe-bel(i=3) for the task of finding the belief in $A$. Trace the algorithm’s performance schematically (show functions, no numbers). What is the time and space complexity of the algorithm?

(d) (10) Apply mbe-mpe(i=3) to find an upper bound for the mpe of the network given $I = 0$. Trace the algorithms. Show how you construct an approximate mpe tuple.

(e) (10) Propose a node duplication simplification for the network that will correspond to the mini-bucket scheme you used in your answer above.

2. (20) Consider the coding networks in the class notes in chapter 8 (figure 8.7),

Figure 1: A Bayesian network
(a) What is the relaxed network, generated by variable duplication that would correspond to the mini-bucket execution of this example. Draw the relaxed network.

(b) Apply schematically, mini-clustering to find the belief of each variable. Show the tree-decomposition over which the mini-clustering algorithm executes.

3. (20) Assume you are given a 4x4 directed grid (like in Figure 1 but having dimension 4 rather than 3).

   (a) Provide an arc-labeled minimal dual graph for the 4x4 grid network.

   (b) Generate a join-graph whose maximal cluster size is 4. Show the functions in each cluster and the variables in each cluster. Label the arcs with the appropriate separators.

   (c) Show the schematic messages that would pass in one iteration of IJGP on your join graph.