ICS 275B
Homework Assignment 3
Spring 2005

Instructor: Rina Dechter Due: Wednesday, April 27

Relevant reading:
Pearl, chapter 4 (4.1-4.3), Bucket-elimination paper, Koller's handout numbers 11, 13, handout on induced-graphs, bucket-tree elimination paper.

1. question 4.1 in Pearl's book.

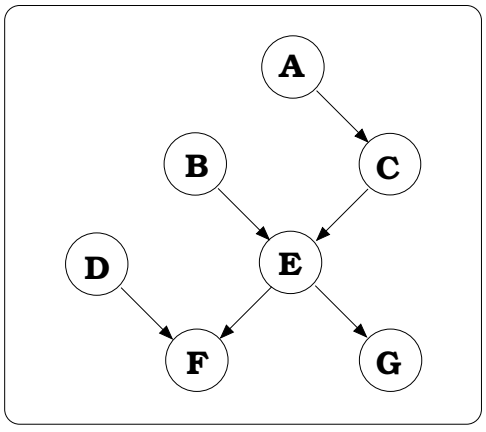
   You can answer this question, either using the poly-tree propagation, or elim-bel or bucket-tree propagation. Explain the network that you are solving.

2. Consider the polytree in Figure below.

   (a) Apply the poly-tree algorithm or the bucket-tree propagation to obtain the marginal probabilities of each variable. You can do calculation by hand or use Hugin or JavaBayes to solve the computational parts of this question.

   (b) Draw the necessary messages and the order in which they are computed.

   (c) Suppose now that the evidence $\{D = 0, C = 1\}$ has been observed. Repeat the calculation to obtain the conditional probabilities of the nodes given this evidence.



a	$p(a)$
0	0.3
1	0.7

b	$p(b)$
0	0.6
1	0.4

e	$p(d)$
0	0.7
1	0.3

a	c	$p(c\|a)$
0	0	0.15
0	1	0.85
1	0	0.25
1	1	0.75

e	g	$p(g\|e)$
0	0	0.10
0	1	0.90
1	0	0.30
1	1	0.70

b	c	e	$p(e\|b,c)$
0	0	0	0.40
0	0	1	0.60
0	1	0	0.45
0	1	1	0.55
1	0	0	0.60
1	0	1	0.40
1	1	0	0.30
1	1	1	0.70

b	c	e	$p(e\|b,c)$
0	0	0	0.25
0	0	1	0.75
0	1	0	0.60
0	1	1	0.40
1	0	0	0.10
1	0	1	0.90
1	1	0	0.20
1	1	1	0.80

Figure 1:

3. Given the directed graph G in Figure 2,

   (a) Compute the induced-graph along ordering: $d_1 = F, C, A, G, D, H, E, B$ and the induced-width for each variable. What is its induced-width of G?

   (b) Use min-induced width to compute an ordered graph, called d_2. Compute the induced-width along d_2.

   (c) What is the induced width of G? Explain your answer.

   (d) Apply elim-bel along the two orderings d_1 and d_2 and show the λ functions created, their placement and the expressions for deriving the functions.

   (e) Show the bucket-tree associated with each ordering and display all the massages (πs and λs) along the tree.

   (f) Assuming you performed all the computation without any evidence. How can you extract the marginal probability of D? Explain.

   (g) Assuming you observed $F = 1$ and $B = 1$, explain how would you would compute (update) $BEL(D)$. Explain.

   (h) Give a bound on the time and space complexity for solving this problem using O notation.

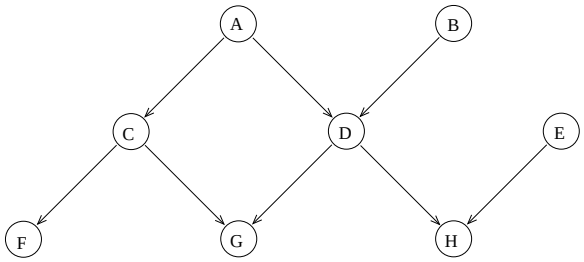


Figure 2: A directed graph

4. (extra credit) Let (G, P) be a Bayesian network and let T be a bucket-tree obtained by applying the bucket-elimination algorithm along an ordering d. Prove that T is an i-map of G.