ICS 275B
Homework Assignment 4
Spring 2005

Instructor: Rina Dechter Due: Wednesday, May 4th

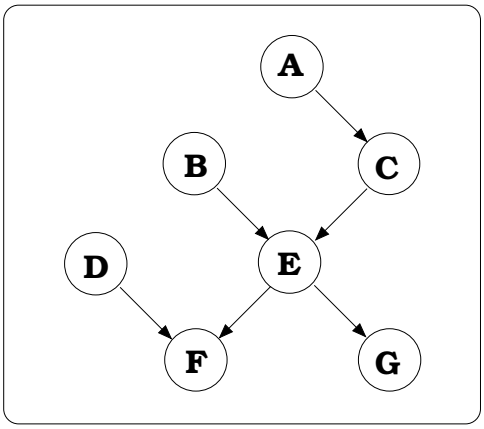
Relevant reading: Pearl chapters 4 and 5, Bucket-elimination paper, Koller's chapter 8, handouts on bucket-tree.

1. question 4.4 in Pearl's book.

2. (optional) question 4.5 in Pearl's book.

3. Solve exercise 4.4 by belief propagation ignoring the loops: Generate the dual join-graph where each function is a single node and apply the message passing (lambdas and pi's) as if the network is singly connected until you reach a stable state. Assess the merit of this technique as a possible approximation method. You can also use REES for this question: belief propagation (IJGP) is one of the methods available. You will only need to provide the network and set the parameters appropriately. Ask Vibhav for more iformation.

4. Consider the polytree in Figure 1. Explain how elim-mpe can find the most probable explanation (mpe) given the evidence, f=0. Demonstrate your computation. You can do calculation by hand or use REES, Hugin or JavaBayes to solve the computational parts of this question.



a	$p(a)$
0	0.3
1	0.7

b	$p(b)$
0	0.6
1	0.4

e	$p(d)$
0	0.7
1	0.3

a	c	$p(c\|a)$
0	0	0.15
0	1	0.85
1	0	0.25
1	1	0.75

e	g	$p(g\|e)$
0	0	0.10
0	1	0.90
1	0	0.30
1	1	0.70

b	c	e	$p(e\|b,c)$
0	0	0	0.40
0	0	1	0.60
0	1	0	0.45
0	1	1	0.55
1	0	0	0.60
1	0	1	0.40
1	1	0	0.30
1	1	1	0.70

d	e	f	$p(f\|d,e)$
0	0	0	0.25
0	0	1	0.75
0	1	0	0.60
0	1	1	0.40
1	0	0	0.10
1	0	1	0.90
1	1	0	0.20
1	1	1	0.80

Figure 1: A polytree.

5. Consider the Bayesian network dag in Figure 2.

   (a) Assume you have evidence over F. Describe how the loop-cutset scheme can find the belief for every variable. What is its time and space complexity.

   (b) Assume you compute the beliefs using join-tree clustering. What would be the time and space complexity. Explain.

   (c) Suggest an efficient scheme for solving the network without recording more than unary functions. Discuss your proposals.

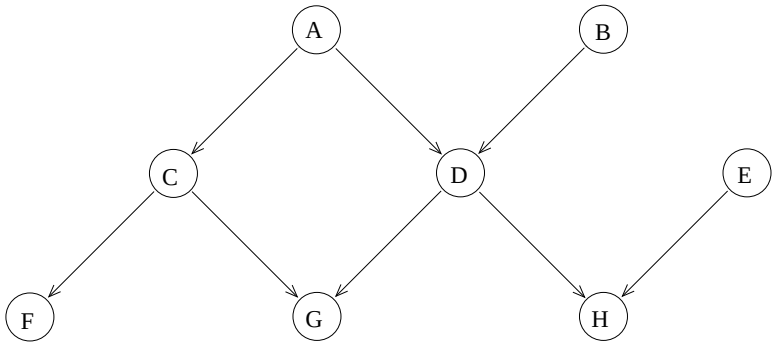


Figure 2: A directed graph

6. Let (G, P) be a Bayesian network and let C be a subset of variables that is a loop-cutset. Prove that $P(C = c)$ can be computed in linear time and space.

7. (extra credit) Which method has better time complexity, the loop-cutset method or join-tree clustering? Prove your claims.

8. Consider the network in Figure 3.

   - What will be the complexity of loop-cutset conditioning on the network. How would it compare with elim-bel? with join-tree clustering?
   - Sometime conditioning on a variable can decompose the graph into two sperate graphs that have no variables in common.
     1. Define a conditioning algorithm that takes advantage of this decomposition property. Provide precise pseudo code for such algorithm.
     2. Show that the running time of the algorithm on the network in Figure 3, with an appropriate choice of conditioning variables is $O(klogk)$.
     3. Suggest an AND/OR search space over which the task of computing the probability of the evidence can be solved.

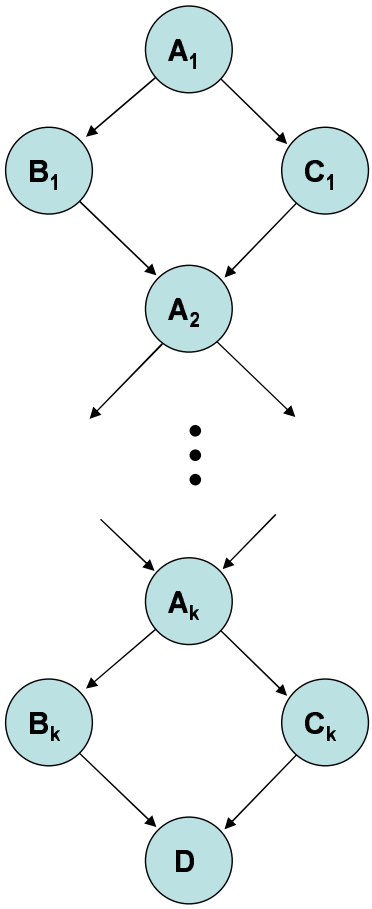


Figure 3: A chain directed graph