Read Lauritzen’s paper and Murphy’s paper. Slides on inference in gaussian models.

1. For Conditional Linear Gaussian (CLG) models, we discussed how to compute the posterior belief at a discrete variable using Variable Elimination.

   (a) Derive a Variable Elimination algorithm to compute posterior belief at ”a” continuous variable. Note that your algorithm should use a single pass. (Hint: Collect Evidence pass in the junction-tree algorithm.)

   (b) What are the possible advantages and disadvantages of the method you derived in part (a) to a two-pass method like the junction-tree algorithm for CLGs.

   (c) What is the time and space complexity of the method you derived in part (a) and how does it compare to the junction-tree algorithm for CLGs.

2. Prove that if we create a junction-tree by eliminating all continuous variables before the discrete variables, the junction-tree must contain at least one strong root.

3. For Figure 1, answer the following:

   (a) Construct a junction-tree for the Bayesian network in Figure 1 by eliminating variables in the order D E F G H A B C.

   (b) Identify the strong-root for the junction tree created in (a).
(c) What is the time and space complexity of junction-tree inference using ordering (a).

(d) Construct a junction-tree for the Bayesian network in Figure 1 by eliminating variables in the order E H A D C B F G.

(e) Does the junction-tree created in (d) have a strong-root. If no, explain why. If yes, identify it.

(f) What is the time and space complexity of junction-tree inference using ordering (d). How does it compare to (a).

4. In practice, a common approach to handle continuous variables is to discretize them. Let us assume that you have developed the following Bayesian network for the problem that your boss wants you to solve. Assume that you are using the bucket-tree algorithm discussed in class to solve the problem. Here, variables $a_i$ and $b_i$ are discrete and take two values {true,false} while $c_i$ can be either discrete or continuous.

(a) Which of the following has better time complexity: (i) $c_i$ is discrete and takes 100 values (ii) $c_i$ is continuous. Explain.

(b) Which of the following has better time complexity: (i) $c_i$ is discrete and takes $2^{10}$ values (ii) $c_i$ is continuous. Explain.