Solutions: Homework 1

ICS 275B
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Problem 1

Let A be the event that you have the disease.
Let B be the event that you tested positive for the disease.
From the data given in the problem, we have the following information.

$$P(A) = 1/10000 = 0.0001$$

$$P(B|A) = 0.99$$

We are asked to find out $P(A|B)$.
From Bayes' theorem, we have:

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

From laws of probability, we know that:

$$P(B) = P(B/A) * P(A) + P(B| \sim A) * P(\sim A)$$

Also,

$$P(B| \sim A) = 0.01$$

$$P(\sim A) = 0.9999$$

Substituting these values in the equation above, we have;

$$P(A|B) = 0.0098126$$

In other words, the information that it is a rare disease reduced our belief that we have the disease to 0.0098126; a very small number. Naturally, we should feel happy.

Problem 2

We will use the following formulae to compute the conditional probabilities.

$$P(x|y, z) = \frac{P(x, y, z)}{P(y, z)}$$

$$P(y|x,z) = \frac{P(x,y,z)}{P(x,z)}$$

$$P(z|x,y) = \frac{P(x,y,z)}{P(x,y)}$$

The conditional probabilities are given below. *1.*

$$P(x=0|y=0,z=0) = \frac{0.12}{0.12+0.09} = 0.5714$$

$$P(x=1|y=0,z=0) = \frac{0.09}{0.12+0.09} = 0.4286$$

$$P(x=0|y=0,z=1) = \frac{0.18}{0.18+0.21} = 0.4615$$

$$P(x=1|y=0,z=1) = \frac{0.21}{0.18+0.21} = 0.5384$$

$$P(x=0|y=1,z=0) = \frac{0.04}{0.04+0.02} = 0.6667$$

$$P(x=1|y=1,z=0) = \frac{0.02}{0.04+0.02} = 0.3333$$

$$P(x=0|y=1,z=1) = \frac{0.16}{0.16+0.18} = 0.4706$$

$$P(x=1|y=1,z=1) = \frac{0.18}{0.16+0.18} = 0.5294$$

2.

$$P(y=0|x=0,z=0) = \frac{0.12}{0.12+0.04} = 0.75$$

$$P(y=1|x=0,z=0) = \frac{0.04}{0.12+0.04} = 0.25$$

$$P(y=0|x=0,z=1) = \frac{0.18}{0.18+0.16} = 0.5294$$

$$P(y=1|x=0,z=1) = \frac{0.16}{0.18+0.16} = 0.4706$$

$$P(y=0|x=1,z=0) = \frac{0.09}{0.09+0.02} = 0.8182$$

$$P(y=1|x=1,z=0) = \frac{0.02}{0.09+0.02} = 0.1818$$

$$P(y=0|x=1,z=1) = \frac{0.21}{0.21+0.18} = 0.5384$$

$$P(y=1|x=1,z=1) = \frac{0.18}{0.21+0.18} = 0.4615$$

3.

$$P(z=0|x=0,y=0) = \frac{0.12}{0.12+0.18} = 0.4$$

$$P(z=1|x=0,y=0) = \frac{0.18}{0.12+0.18} = 0.6$$
$$P(z=0|x=0,y=1) = \frac{0.04}{0.04+0.16} = 0.2$$
$$P(z=1|x=0,y=1) = \frac{0.16}{0.04+0.16} = 0.8$$
$$P(z=0|x=1,y=0) = \frac{0.09}{0.09+0.21} = 0.3$$
$$P(z=1|x=1,y=0) = \frac{0.21}{0.09+0.21} = 0.7$$
$$P(z=0|x=1,y=1) = \frac{0.02}{0.02+0.18} = 0.1$$
$$P(z=1|x=1,y=1) = \frac{0.18}{0.02+0.18} = 0.9$$

Problem 2.1 Pearl's book

Let U_1,U_2 and U_3 be the events that Urns 1,2 and 3 are selected respectively.
Let B be the event of drawing a black ball and W be the event of drawing a White ball.
a. Here we have to find $P(B,U_2) = P(B|U_2) * P(U_2)$.
From data, we have $P(B|U_2) = \frac{2}{6}$ and $P(U_2) = \frac{1}{3}$.
So, $P(B,U_2) = \frac{1}{9}$.
b. The probability of drawing a black ball is given as follows.
$P(B) = \sum_i P(B,U_i) = \sum_i P(B|U_i) * P(U_i)$
$P(B) = \frac{1}{3} * \frac{1}{3} + \frac{1}{2} * \frac{1}{3} + \frac{2}{3} * \frac{1}{3}$
$P(B) = \frac{1}{2}$.
c. Here we have to compute $P(U_2|B)$.
By Bayes' theorem, $P(U_2|B) = \frac{P(B|U_2)*P(U_2)}{\sum_i P(B|U_i)*P(U_i)}$.
$P(U_2|B) = \frac{\frac{1}{3}*\frac{1}{3}}{\frac{1}{2}}$.
$P(U_2|B) = \frac{2}{9}$.

Problem 2.3a Pearl's book

$P(H_i|e_1^1, e_3^2) = \alpha * P(e_1^1|H_i) * P(e_3^2|H_i) * P(H_i)$
$P(H_i|e_1^1, e_3^2) = \alpha*(0.5, 0.06, 0.5, 1.0)*(0.1, 0.1, 0.8, 0.05)*(0.099, 0.009, 0.001, 0.891)$
$P(H_i|e_1^1, e_3^2) = \alpha * (0.00495, 0.000054, 0.0004, 0.4455)$.
$P(H_i|e_1^1, e_3^2) = (0.099, 0.0011, 0.008, 0.892)$.

from which we conclude that the chance of an attempted burglary (H_2 or H_3) is $0.0011 + 0.008 = 0.0091 = 0.91\%$.

Problem 4

The set of numbers in (ii) is sufficient for calculating $P(H|E_1, E_2)$.
It can be proved that it is sufficient by Bayes' formula which is stated below:
$P(H|E_1, E_2) = \frac{P(E_1,E_2|H)*P(H)}{P(E_1,E_2)}$.
$P(E_1, E_2)$ is not necessary as it can be considered as a normalization constant. Note that to normalize we should have the information that the events are exhaustive and mutually exclusive.

The information in (i) is not sufficient because according to chain rule and Bayes' theorem, we should have the following information.
$P(H|E_1, E_2) = \frac{P(H,E_1,E_2)}{P(E_1,E_2)}$.
$P(H|E_1, E_2) = \frac{P(E_1|H,E_2)*P(E_2|H)*P(H)}{P(E_1,E_2)}$.
Clearly, the information given in (i) is not sufficient to compute $P(E_1|H, E_2)$.

The information given in (iii) is a subset of the information given in (i) and since (i) is not sufficient, we conclude that the information given in (iii) is also not sufficient.