COMPSCI 276
Homework Assignment 5
Spring 2017

Instructor: Rina Dechter — Due:Monday, May 15th

Relevant reading: Dechter chapter 5, 7.1 and Darwiche chapter 7, 8.1

1. (Extra credit, 5 pts). Read Dechter chapters 4 and 5 and provide comments on clarity and typos.

2. (30) Given the directed graph G in Figure 1 (used also in homework 3),

   (a) Show the bucket-tree associated with the ordering $d_1 = F, C, A, G, D, H, E, B$ and display all the messages (πs and λs) along the tree.

   (b) Assuming you performed all the computation without any evidence. How can you extract the marginal probability of D? Explain.

   (c) Assuming you observed $F = 1$ and $B = 1$, explain how you would compute (update) $BEL(D) = P(D|F = 1, B = 1)$.

   (d) Give a bound on the time and space complexity for solving this problem using O notation.

   (e) Assume you have evidence over F. Describe how the loop-cutset scheme can find the belief for every variable. What is its time and space complexity?

   (f) Assume you compute the beliefs using join-tree clustering. What would be the time and space complexity? Explain.

   (g) Suggest an efficient scheme for solving the network without recording more than unary functions. Discuss your proposals.

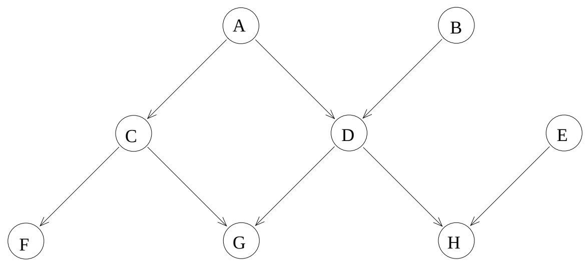

Figure 1: A directed graph

3. (10) Consider the network in Figure 2. Find a join-tree representation for the network and show how you would compute $Bel(D_1|M_2 = false, M_3 = true, M_4 = false)$ schematically (demonstrating the type of messages that would be passed).

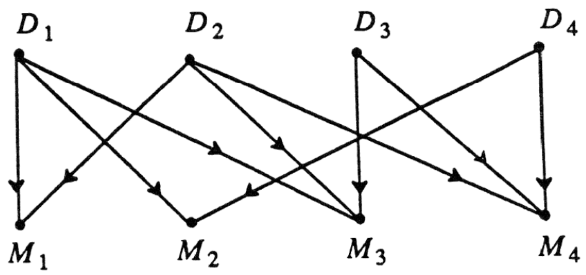

Figure 2: A two layer network

4. (10) Consider the network in Figure 2.

   (a) What is the dual graph of this network?

   (b) What is a minimal dual graph? Is the problem acyclic?

   (c) If we have evidence that $D_1 = true$, is the network conditioned on this evidence, acyclic?

5. (10) (extra credit) Let (G, P) be a Bayesian network where G is an directed acyclic graphs over variables X and let $C \subseteq X$ be a subset of variables that form a loop-cutset. Prove that $P(C = c)$ can be computed in linear time and space.

6. (10) (extra credit) Which method has better time complexity, the loop-cutset method or join-tree clustering? Prove your claims.

7. (10) Consider the network in Figure 3.

   (a) What will be the complexity of loop-cutset conditioning on the network?

   (b) How would it compare with BE-bel?

   (c) How would it compare with join-tree clustering?

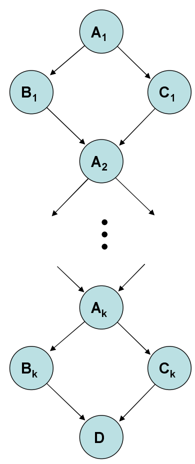

Figure 3: A chain directed graph

8. (extra credit) Prove that: Given an acyclic graphical model, algorithm BTE can compute the marginal problem on each scope of an input function in linear time and space.

9. Consider the Bayes network DAG in Figure 4:

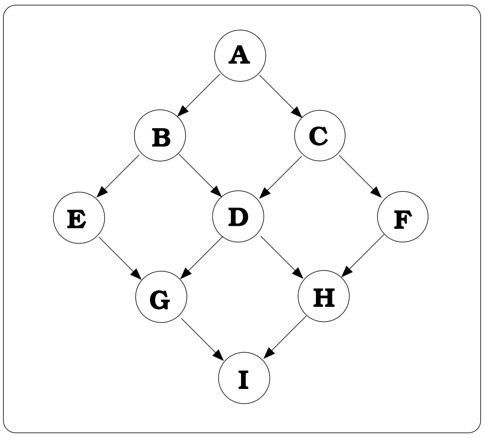

Figure 4: A Bayesian network

   (a) (5) Suppose you want to compute the probability of evidence. What would be the time and space complexity of doing so using Bucket-elimination? a. when the evidence is on variable E? b. when the evidence is I? c. when you want to compute the belief of $P(A|I = 0)$?

   (b) (5) Assume that you have a Markov network grid when the potential are pair-wise (same as the above figure but remove the arrows). Answer the above question relative to such a grid markov network.

   (c) (10) Assume you want to solve the problem in Figure 4 using algorithm cycle-cutset conditioning, what is the smallest cycle-cutset you can find (1-cutset) relative to the moral graph? What is the smallest 2-cutset you can find?

   (d) (10) Describe how would 2-cutset conditioning can be applied to this problem. What would be the time and memory of 2-cutset conditioning on this problem?

   (e) (10) Find and present a pseudo tree of the network in Figure 4 whose depth is minimal as best you can. Call this tree T_1.

   (f) (extra credit, 5) Generate an AND/OR search tree driven by T_1 assuming each variable has at most two values. Ignore the weights.

10. (10) Consider the graph in Figure 5. Find a 1-cutset and a 2-cutset and a 3-cutset for the problem. Discuss how would cutset-conditioning algorithm work for each one of these cutset cases. provide time and space analysis.

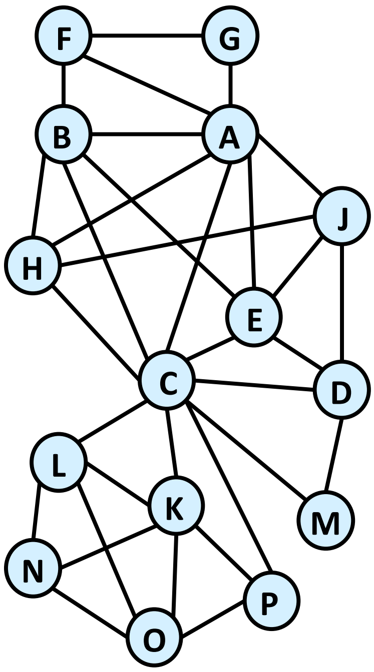

Figure 5: A graph