ICS 6A
Solution to Homework Assignment 4
Winter 2004

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Answer the following questions (explain your answers).

   Answer: \[
   \frac{\binom{102}{2} \cdot \binom{2}{2}}{\binom{102}{4}} = 0.0075
   \]

   Answer: The number of positive integers not exceeding 100 is 100, and they are 1, 2,...100. The number of positive integers not exceeding 100 and are divisible by 3 is 33, and they are 3 \times 1, 3 \times 2,...3 \times 33. So the probability is \[
   \frac{33}{100} = 0.33
   \]

   Answer:
   - the probability of rolling a total of 8 when two dice are rolled: \[
   \frac{5}{36} = 0.139. \ (Because \ 8 = 6+2 = 5+3 = 4+4 = 3+5 = 2+6 \rightarrow 5 \ possibilities)
   \]
   - the probability of rolling a total of 8 when three dice are rolled: \[
   \frac{21}{216} = 0.097. \ (Because \ 8 = 1+1+6 = 1+2+5 = ... \rightarrow 21 \ possibilities)
   \]

0.139 > 0.097, so rolling a total of 8 when two dice are rolled is more likely.

   Answer: Assume the probability of each outcome except “3” is \( p \), then the probability of the outcome “3” is \( 2p \). So \( 5 \times p + 2p = 1 \implies p = \frac{1}{7} \). The probability of the outcome “3” is \( 2p = \frac{2}{7} \), the probability of each other outcome except “3” is \( \frac{1}{7} \).

5. Rosen, page 376, problem 5. There are 6 pairs that some to 7. All but one (4,3) have probability 1/49. (4,3) has probability 4/49. The total probability is therefore 9/49.

   Proof: From THEOREM 2 on page 264, we have:
   \[
   P(E \cup F) = P(E) + P(F) - P(E \cap F) \implies P(E \cap F) = P(E) + P(F) - P(E \cup F) = 0.8 + 0.6 - P(E \cup F).
   \]
   For any probability, it cannot exceed 1, so \( P(E \cup F) \leq 1 \), such that:
   \[
   P(E \cap F) \geq P(E) + P(F) - 1 = 0.8 + 0.6 - 1 = 0.4.
   \]
   Finally, we have \( P(E \cap F) \geq 0.4 \)
   From the definition \( P(E \cap F) \geq P(E) \). Since \( P(E) = 0.8 \) we get that \( P(E \cap F) \geq 0.8 \).

   The probability is 1/16.

   \[
   P(E) = 1/2, \ P(F) = 1/2 \ and \ P(E \cap F) = 1/4. \ Since \ P(E \cap F) = P(E) \cdot P(F) \ E \ and \ F \ are \ independent.
   \]

   a) exactly three boys: \( C(5,3) \times 0.51^3 \times (1-0.51)^{5-3} = 0.318 \)
   b) at least one boy: \( 1 - (1-0.51)^5 = 0.972 \)
   c) at least one girl: \( 1 - 0.51^5 = 0.965 \)
   d) all children of the same sex: \( 0.51^5 + (1 - 0.51)^5 = 0.063 \)
   a) the probability of no successes: \((1 - p)^n\)
   b) the probability of at least one success: \(1 - \text{result of } a = 1 - (1 - p)^n\)
   c) the probability of at most one success: \((1-p)^n + C(n, 1) \cdot p \cdot (1-p)^{(n-1)} = (1-p)^n + n \cdot p \cdot (1-p)^{(n-1)}\)
   d) the probability of at least two successes: \(1 - \text{result of } c = 1 - (1 - p)^n - n \cdot p \cdot (1-p)^{(n-1)}\)