From Exact to Anytime Solutions for Marginal MAP

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Introduction

- **Marginal MAP**
  - Mode of probability distribution after marginalizing subset of variables
  - Complexity Class: $\text{NP}^{\text{PP}}$ Complete
    - MPE (NP-Complete): optimizing over max variables
    - PR (#P-Complete): evaluating full instantiation

- **Application to Probabilistic Planning**
  - Marginal MAP query returns optimal probabilistic conformant plan*

* “Applying Search Based Probabilistic Inference Algorithms to Probabilistic Conformant Planning: Preliminary Results”, 2016 ISAIM
## Earlier Works on Marginal MAP Inference

### Earlier Approaches

<table>
<thead>
<tr>
<th>Approach</th>
<th>Description</th>
</tr>
</thead>
</table>
| [Park & Darwiche 2003] | • Exact Solution  
• Depth First Branch and Bound  
with Dynamic Variable Ordering  
• Join-tree upper bound  
Relax ordering  
Systematic Search Algorithm |
| [Yuan & Hansen 2009] | • Exact Solution  
• Depth First Branch and Bound  
with Static Variable Ordering  
• Incremental Join-tree upper bound  
Reduced heuristic computation time |
| [Marinescu, Dechter, Ihler 2014] | • Exact Solution  
• AND/OR Branch and Bound  
• WMB + Cost shifting schemes  
Stronger Heuristic  
Compacter AND/OR Search Space |
| [Marinescu, Dechter, Ihler 2014] | • Exact Solution  
• AND/OR Best First  
• AND/OR Recursive Best First  
Best First Based Search Strategy  
Avoid Solving Summation Problems |

- [Liu, Ihler 2013] Variational algorithms
- [Maua, De Campos 2012] Factor-set elimination algorithm

### Motivation

- Best First Schemes avoid evaluating summation sub problems, but they requires enormous amount of memory ➔ Turn to anytime approach
Probabilistic Graphical Models

- A graphical model ($\mathbf{X}, \mathbf{D}, \mathbf{F}$)
  - $\mathbf{X} = \{X_1, \ldots, X_n\}$ variables
  - $\mathbf{D} = \{D_1, \ldots, D_n\}$ domains
  - $\mathbf{F} = \{f_1, \ldots, f_m\}$ functions

- Operators
  - Combination (product)
  - Elimination (max/sum)

- Tasks
  - Probability of Evidence (PR)
    \[
    Pr(e) = \sum_{X_s} \prod_j f_j(X_s, e)
    \]
  - Most Probable Explanation (MPE)
    \[
    x_{MPE} = \arg \max_x \prod_j f_j(x)
    \]
  - Marginal MAP (Maximum A Posteriori)
    \[
    x_{MMAP} = \arg \max_{x_m \in X_M} \sum_{X_s \in X_s} \prod_j f_j(x_m, x_s)
    \]

All these tasks are NP-hard

Exploit problem structure (primal graph)
AND/OR Search Space for MMAP

- **Constrained variable ordering**
- **Primal graph**
  \[ X_M = \{A, B, C, D\} \]
  \[ X_S = \{E, F, G, H\} \]
- **Constrained pseudo tree as backbone**
  - Merge identical sub-problems
  - (Conditional independence)
Anytime AND/OR Search for MMAP

- **Anytime AOBB (BRAOBB)**

  Depth First Branch and Bound (AOBB)

  Breadth Rotate AOBB

  Prune node n if current best solution is better than optimistic evaluation at n

  Problem decomposition rejects anytime performance of AOBB
  Rotate through sub-problems
Anytime AND/OR Search for MMAP

- Weighted Best First Search
  - AND/OR Best First
  - Expand Nodes with best heuristic evaluation value $f(n)$

- Weighted Best First Search
  - Initialize $w$
  - While $w \geq 1$
    - Inflate heuristic by $w$
    - AOBF (sub-optimal solution within $w$)
    - Optionally Revise traversed search space
    - Reduce $w$

- Weighted Restarting AOBF (WAOBF)
- Weighted Restarting RBFAOO (WRBFAOO)
- Weighted Repairing AOBF (WRAOBF)
Experiment Setup

Benchmark Instances

<table>
<thead>
<tr>
<th>Domain</th>
<th># instances</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRID</td>
<td>75</td>
</tr>
<tr>
<td>PEDIGREE</td>
<td>50</td>
</tr>
<tr>
<td>PROMEDAS</td>
<td>50</td>
</tr>
</tbody>
</table>

Problem instances are modified from PASCAL2 Probabilistic Inference Challenge Data Set (http://www.cs.huji.ac.il/project/PASCAL/)

Algorithm Parameters

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameters</th>
<th>Memory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted Mini Bucket Heuristic</td>
<td>i-bound from 2 to 20</td>
<td>-</td>
</tr>
<tr>
<td>BRAOBB</td>
<td>Rotation Limit 1000</td>
<td>Max 24 GB</td>
</tr>
<tr>
<td>WAOBF / WRAOBF / WRBFAOO</td>
<td>Starting Weight 64</td>
<td>Max 24 GB, Cache 4 GB</td>
</tr>
</tbody>
</table>

Performance Measures

- Responsiveness, Quality Score
## Performance Regimes

<table>
<thead>
<tr>
<th>AND/OR Search for MMAP</th>
<th>Overall</th>
<th>Pedigree</th>
<th>Promedas</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Resp.</td>
<td>Quality</td>
<td>Resp.</td>
</tr>
<tr>
<td>Exact</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AOBB</td>
<td>89%</td>
<td>339%</td>
<td>84%</td>
</tr>
<tr>
<td>AOBF</td>
<td>50%</td>
<td>208%</td>
<td>42%</td>
</tr>
<tr>
<td>RBFAOO</td>
<td>58%</td>
<td>90%</td>
<td>42%</td>
</tr>
<tr>
<td>Anytime</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>WAOBF</td>
<td>82%</td>
<td>365%</td>
<td>88%</td>
</tr>
<tr>
<td>WRBFAOO</td>
<td>86%</td>
<td>394%</td>
<td>90%</td>
</tr>
<tr>
<td>WRAOBF</td>
<td>82%</td>
<td>339%</td>
<td>88%</td>
</tr>
<tr>
<td>BRAOBB</td>
<td>86%</td>
<td>365%</td>
<td>58%</td>
</tr>
</tbody>
</table>

- Summarized from 1 hour time bound,
- Responsiveness: WMB-MM(18), Quality Score: WMB-MM(12) heuristic

- WRBFAOO is the overall best performed algorithm
- BRAOBB is the second best performer, but the best at PROMEDAS DOMAIN
WRBFAOO vs. BRAOBB

- Closer look at individual problem instances

- Each point \((W_c, W_s)\) represents difficulty of problem

- Time/ Memory Complexity is Exponential in \(W\)

- Easy Problems: \(W_c < 60\)

- Harder Problems: \(200 < W_c, 10 < W_s\)

- \(60 < W_c < 200, W_s < 10\)
Conclusion

- Improvement from Exact to Anytime
  - Anytime Best-First approach
    - Recovers responsiveness close to Depth-First schemes
    - Provide high quality solutions

- Future Work
  - Better Search Strategy
    - Memory issue with hard problems (Ws > 10, Wc > 200)
  - Integrate approximation for summation problems
    - From exact to approximation