**INTRODUCTION**

Best-first search can be regarded as an anytime scheme for producing lower bounds on the optimal solution, a characteristic that is mostly overlooked. We explore this topic in the context of AND/OR best-first search (ABOP), guided by the mini-bucket elimination (MBE) heuristic, when solving graphical models. In that context, the impact of the secondary heuristic for subproblem ordering may be significant, especially in the anytime context.

We illustrate in this paper that the new concept of bucket errors can advise in providing effective subproblem orderings in AND/OR search.

**BACKGROUND**

**Graphical Model**
- \( M = (X, D, F) \)
- \( X = \{ X_i : i \in V \} \), a set of variables indexed by a set \( V \)
- \( D = \{ D_i : i \in D \} \) is the set of finite domains of values for each \( X_i \)
- \( F = \{ f(X_i : x) \in F \} \) is a set of discrete functions, where \( S \subseteq V \) and \( X_S \subseteq X \) is the scope of \( f_S \)

**Bucket Elimination (BE)** [Dechter 1999]
- Solves the mini-sum problem by eliminating variables one at a time.
- Complexity: exponential in the induced (tree) width of the underlying primal graph

**Mini-Bucket Elimination (MBE)** [Dechter and Rish 2003]
- Can approximate BE by selecting a relaxation ordered by duplicating variables to bound the treewidth by a parameter known as the \( \lambda \)-bound.

**AND/OR Search**
- The AND/OR search space of a graphical model is guided by the decomposition suggested by the pseudo-tree.

**Bucket Error** [Dechter et al. 2015]
- The difference between the function-message that would have been computed in an individual bucket without partitioning and the message computed by MBE.
- Example using BE and MBE figure above: \( \text{Err}(A, B) = X^*(A, B) - (X^*(A) - X^*(B)) \)
- Captures the local error induced by the MBE partitioning
- Equivalent to depth-1 lookahead residual

** IMPACT OF SUBPROBLEM ORDERING **
- Given \( f(t) < C^* \), the selection of which subproblem to expand next \( f_t \) can influence the number of nodes expanded.
- Intuitively, we want to expand the subproblem that leads to the greatest increase in \( f_t \) therefore discovering as soon as possible that the optimal extension to \( t \) is >\( C^* \)
- The increase is equivalent to the look-ahead residual, making bucket errors a natural choice to build an informed \( f_t \)
- Example:

**EXPERIMENTS AND CONCLUSION**

We ran AND/OR Best-First (ABOP) search using the MBE-MM heuristic [Ibler et al. 2012]
- Tested various \( \lambda \)-bounds to vary heuristic strength
- Compare 4 variants against the ad-hoc baseline
- Ranked the anytime performance on a normalized time scale over benchmarks

**REFERENCES**


