Frequencies
Outline

- Convolution
- Resizing
- Frequency analysis
- HW2
Recall

A Gaussian kernel gives less weight to pixels further from the center of the window. This kernel is an approximation of a Gaussian function:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 90 & 90 & 90 & 90 & 0 & 0 & 0 \\
0 & 0 & 90 & 90 & 90 & 90 & 0 & 0 & 0 \\
0 & 0 & 90 & 90 & 90 & 90 & 0 & 0 & 0 \\
0 & 0 & 90 & 0 & 90 & 90 & 0 & 0 & 0 \\
0 & 0 & 90 & 90 & 90 & 90 & 0 & 0 & 0 \\
0 & 0 & 90 & 90 & 90 & 90 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
H[i,j]
\]

\[
F[i,j]
\]
Convolution vs correlation

(1-d)

\[ G[i] = F[i] \ast H[i] = \sum_u F[u] H[i - u] \quad \text{(convolution)} \]

\[ = H[i] \ast F[i] = \sum_u H[u] F[i - u] \quad \text{(commutative property)} \]

\[ G[i] = F[i] \otimes H[i] = \sum_u H[u] F[i + u] \quad \text{(cross-correlation)} \]

\[ = F[i] \ast H[-i] \quad \text{(exercise for reader!)} \]
Convolution vs correlation (2-d)

Convolution:

\[ G[i, j] = F \ast H = H \ast F = \sum_u \sum_v H[u, v] F[i - u, j - v] \]

Correlation:

\[ G[i, j] = F \otimes H = \sum_u \sum_v H[u, v] F[i + u, j + v] \]

Matlab >> conv2(H,F)

How can one use convolution to perform correlation?
Centered filters

\[ G[i, j] = F \ast H = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u, v]F[i + u, j + v] \]

What happens if filter origin is at corner (not the center)?
Gaussian filter

\[ G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}} \]

Standard deviation \( \sigma \): determines extent of smoothing

Matlab:

```matlab
>> G = FSPECIAL('gaussian',HSIZE,SIGMA)
>> imagesc(G);
>> surf(G);
```
Denoising

Additive Gaussian Noise

Gaussian Filter
Salt and pepper noise
Alternative idea: Median filtering

A **median filter** operates over a window by selecting the median intensity in the window.
Median filter

What advantage does median filtering have over Gaussian filtering?

• Robustness to outliers

Source: K. Grauman
Median filter

Salt-and-pepper noise

Median filtered

MATLAB: medfilt2(image, [h w])

Is this a linear shift-invariant operator?

Source: M. Hebert
Outline

• Convolution
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Image sub-sampling

1/2  1/4  (2x zoom)  1/8  (4x zoom)

Aliasing! What do we do?
Examples of GOOD sampling
Examples of BAD sampling -> Aliasing
Gaussian pre-filtering

Solution: filter the image, \textit{then} subsample
Gaussian stacks (HW2)

- In the original image space, filter size (standard deviation) should double for each size reduction. Why?
Compare with...

1/2  1/4  (2x zoom)  1/8  (4x zoom)

Slide by Steve Seitz
Gaussian (lowpass) pre-filtering

Solution: filter the image, then subsample

- How can we speed up?
Image Pyramids

Idea: Represent N x N image as a “pyramid” of 1 x 1, 2 x 2, 4 x 4, ..., 2^k x 2^k images (assuming N = 2^k)

Known as a **Gaussian Pyramid** [Burt and Adelson, 1983]
- In computer graphics, a *mip map* [Williams, 1983]
- A precursor to *wavelet transform*

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