Grouping
Gestalt: The whole versus the part

• Whole is greater than sum of its parts

• Relationships among parts can yield new properties/features

• Psychologists identified series of factors that predispose set of elements to be grouped (by human visual system)

• Open research: How to operationalize into computer vision algorithms.
Gestalt factors

- Not grouped
- Proximity
- Similarity
- Similarity
- Common Fate
- Common Region
Parallelism

Symmetry

Continuity

Closure
K-means clustering using intensity alone and color alone

Grouping by similarity

Image | Clusters on intensity | Clusters on color
K-means using color alone, 11 segments
K-means using color alone, 11 segments.
Grouping by similarity and proximity

K-means using colour and position, 20 segments
What happens if clusters are not circular?
Populate grid with “blobs” instead of points

Find modes of height field with mean-shift algorithm
Fig. 6.2. Various steps of ICM
Fig. 2.3. Smoothing with the wrong prior. (a) Original, (b) degraded image, (c) MAP estimate $\beta = 1$, (d) MAP estimate $\beta = 0.3$, (e)
Interactive segmentation

(c) Cardiac MR
(d) LV Segment
(e) Lung CT
(f) Lobe Segment
(g) Liver MR
(h) Liver Segment
* Max flow you can push through the network = min cut
  -- ie capacity of cut with smallest total capacity
* Links saturated by max flow = links separated by min cut
**Example: optimization as graph cut problem**

**Problem:** \( \min_x E = f_1(x_1) + f_2(x_2) + g(x_1, x_2) \)

where: \( f_1(0)=7; f_1(1)=2; f_2(0)=3; f_2(1)=5; \) \[\text{[all weights +ve]}\]
\( g(0,1)=g(1,0)=1; \quad g(0,0)=g(1,1)=0; \) \[\text{[canonical form]}\]

**Graph:**

\[\text{source } x=1\]

\[\text{sink } x=0\]

- \( f_1(0)=7; f_1(1)=2; f_2(0)=3; f_2(1)=5; \)
- \( g(0,1)=g(1,0)=1; \quad g(0,0)=g(1,1)=0; \) \[\text{[canonical form]}\]

**Trial Solution** \( x_1=1, x_2=0 \)

**Energy:** \( E(x_1, x_2) = 7 + 1 + 5 = 13 \)
Example: graph cut optimization – min cut/max flow

Solve (exactly) by augmenting flow:

Solution from saturated paths

\[ x_1=0, \ x_2=1 \quad E(x_1, x_2) = 2 + 1 + 3 = 6 \]
“Smart scissors”

“GrabCut” — Interactive Foreground Extraction using Iterated Graph Cuts

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Microsoft Research Cambridge, UK

Vladimir Kolmogorov

Andrew Blake

Figure 1: Three examples of GrabCut. The user drags a rectangle loosely around an object. The object is then extracted automatically.
The problem with MinCut
Graph Terminology

Similarity matrix $S = [S_{ij}]$
is generalized adjacency matrix
Graph Terminology

Degree of node: \[ d_i = \sum_{j} S_{ij} \]
Graph Terminology

Volume of set:

\[ \text{vol}(A) = \sum_{i \in A} d_i, \ A \subseteq V \]
Cuts in a graph

\[ \text{cut}(A, \bar{A}) = \sum_{i \in A, j \in \bar{A}} S_{i,j} \]
Normalize cuts in a graph

- (edge) $N_{cut} = \text{balanced cut}$

\[
N_{cut}(A, B) = \text{cut}(A, B) \left( \frac{1}{\text{vol}(A)} + \frac{1}{\text{vol}(B)} \right)
\]
Finding high quality cuts

• Problem: finding the minimum normalized cut is NP-hard, even for k=2.

• Solution: Formulate as a quadratic program and relax integer constraints to get an approximate solution
  – uses nice ideas from linear algebra and spectral graph theory
Laplacian matrix D-W

Let $x = X(1,:)$ be the indicator of group 1

$$asso(A, A) = x^T W x$$
Laplacian matrix $D-W$

$$\text{Cut}(A, V-A) = x^T Dx - x^T W x$$

$$\text{vol}(A) - \text{asso}(A,A)$$
$Ncut(X) = \frac{1}{K} \sum_{l=1}^{K} \frac{cut(V_l, V - V_l)}{vol(V_l)}$

$= \frac{1}{K} \sum_{l=1}^{K} X_l^T (D - W) X_l \frac{X_l^T D X_l}{X_l^T D X_l}$

$X \in \{0, 1\}^{N \times K}, X 1_K = 1_N$
becomes

\[ N_{cut}(Z) = \frac{1}{K} tr(Z^T W Z) \quad Z^T D Z = I_K \]

Rayleigh and Ritz Says:

Eigensolutions

\[ (D - W) z^* = \lambda D z^* \]

\[ Z^* = [z_1^*, z_2^*, ..., z_k^*] \]