HW1

1. (RV 2.3,2.4)

   (a) Suppose that an image is created by a camera in a certain world. Now imagine the same camera placed in a similar world in which everything is twice as large and all distances between objects have also doubled. Compare the new image and with one formed in the original world. Assume perspective projection.

   (b) Suppose that an image is created by a camera in a certain world. Now imagine the same camera placed in a similar world in which everything has half the reflectance and the incident light has doubled. Compare the new image and with one formed in the original world. Ignore inter-reflections - illumination of one part of the scene by light reflected from another.

2. (RV 2.5). (a) Show that in a properly focused imaing system $f' = (1+m)f$, where f' is the distance from the lense to the image plane, and f is the focal length of the lens. f' is called the effective focal length. (b) Show that the distance between the image plane and an object must be

$$(m + 2 + \frac{1}{m})f$$

   (c) How far must the object be from the lense for unit magnification?

3. (RV 2.6) What is the focal length of a compound lense obtained by planing two thin lenses of focal length f_1 and f_2 against one another? Hint1: Recall that all rays passing through the lens that are parallel to the optical axis will be focused together at point f units away on the optical axis. Since lenses are symmetric, rays emanating from an object f units away will emerge parallel to the optical axis after passing through a lens. Use this fact to explain why an object at a distance f_1 on one side of the compond lense will be focused at a distance f_2 on the other side.

4. (MA-4.2) (Radiance transport)

   (a) We derived in class the fact that radiance is conserved along straight line paths in a non-absorbing medium. Show that irradidance is not conserved along straight line paths. This is one motivation for using radiance L as base quantity for measuring light.

(b) Assume we in an absorbing medium that radiance travelling along a small straight line of length ∂x decreases from N to $N - (\alpha \partial x)N$. Write down the expression for radiance transferred from one patch to another in the medium as a function of α, the absorption constant.

5. (Spherical coordinates)

   (a) Derive the fact that the projected surface area of a unit-area patch, integrated over the viewing hemisphere, is π.

$$\int_{2\pi} cos\theta \partial\omega = \pi$$

   (b) (MA-4.12) Consider an observor looking at a sphere of radius ϵ whose center is distance z away from the observor. Show that , for $z \gg \epsilon$, the solid angle subtended by the sphere is approximately

$$\pi(\frac{\epsilon}{z})^2$$

6. Image formation. Recall we can relate irradiance arriving at an image pixel E to radiance leaving a scene patch L as

$$E = L\frac{\pi}{4}(\frac{d}{f})^2 \cos^4 \alpha$$

where d is the diameter of the lens, f is its focal length, z is the distance of the scene patch to the lens, and α is the angle between the optical axis and a vector connecting the center of the lens and patch.

   (a) Show that the angle subtended by the lens, as seen from the patch, is

$$\frac{\pi}{4}(\frac{d}{z})^2 \cos^3 \alpha$$

   (b) (RV-10.2) Explain why the irradiance arriving at an image pixel is not a function of the distance to the patch. After all, when the lens is twice as far from the point, it collects only one-quarter as much light emitted from the point.

   (c) (RV-10.2) Explain why the irradiance is not a function of the orientation of the patch. After all, when the patch is oriented away from the lens, its apparant surface area is reduced due to foreshortening.