CS 217 - Light & Geometry in Computer Vision (Prof. Deva Ramanan)

Lecture 12 - Monday,April 27, 2009 (Scribe : Ankit Gupta)

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Shape from shading

We are trying to estimate the surface height z(x,y) of an object from a single image of the object.

1 Assuming the point light source to be far away from the object

Image intensity at any point (x,y) of the image ,

I(x,y) = $\rho(x,y)N(x,y)^T.S$

where $\rho(x,y)$= albedo , N(x,y)=normal at position (x,y) and S is the direction of the point light source.

In ideal case $\rho(x,y)$=1 and ||S||=1.

So,

I(x,y) = $\{\frac{-p,-q,-1}{\sqrt{1+p^2+q^2}}\}.\{\frac{-p_s,-q_s,-1}{\sqrt{1+p_s^2+q_s^2}}\} = \frac{p_s p+q_s q+1}{\{\sqrt{1+p^2+q^2}\}\{\sqrt{1+p_s^2+q_s^2}\}}$ =R$_s(p,q)$ { Reflectance Map}

2 Estimation from one Image

f(p(x,y),q(x,y)) = A+ B+ C

we need to find p and q which minimizes 'f'

A=$\iint$ {R(p(x,y),q(x,y) - I(x,y)}$^2 \partial x \partial y$

we will not find a unique solution for A as there may be a family of pixels with the same I(x,y)

Also , there is no unique solution for I(x,y) $<$ 1

B= $\iint p_x(x,y)^2 + p_y(x,y)^2 + q_x(x,y)^2 + q_y(x,y)^2 \partial x \partial y$

where p$_x(x,y)$ = p(x+1,y)-p(x,y) and so on

C=$\iint \{p_y(x,y) - q_x(x,y)\}^2 \partial x \partial y$

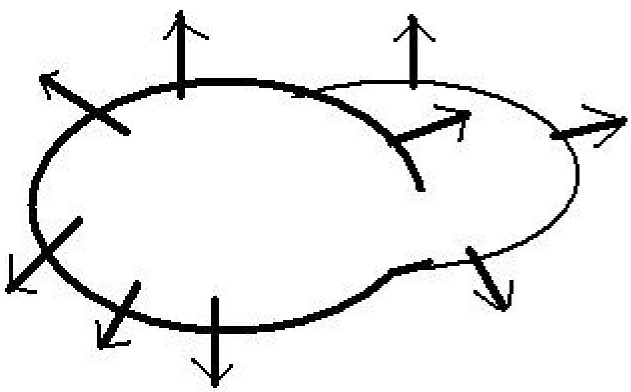
How to minimize 'f' ?

- we can use calculus of variations to determine the Euler's equation (equivalent to $\min_x f(x) \equiv \partial f(x)/\partial x = 0$)
- Discretize the functional and apply Non linear optimization (e.g - conjugate grading algorithm or gradient descent)

3 Occluding Contour

Defn : Points on the surface whose Normal swings away from the camera (parallel to image plane)

What info can be extracted from object boundaries ?



We know that tangent plane to a surface includes

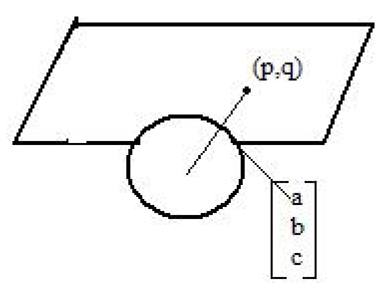
1. Tangent to boundary
2. Vector Pointing to Camera

we can write down N(x,y) for these points .

3.1 Projection

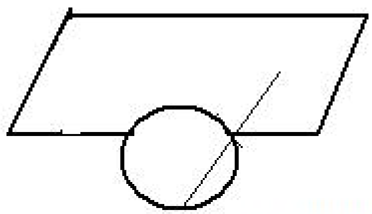
3.1.1 Gnomic Projection

Shift of origin of the Gaussian Sphere from center to the south pole



Conventional Projection [a b c]^T= $(-p, -q, 1)^T / \sqrt{p^2 + q^2 + 1}$

Problem :The Normal [a b c]^{T}maps to the point (p,q) on the plane, so we have numerically unstable mapping for points on the occluding boundary. This problem is removed in Gnomic projection.



Origin at South Pole

What does "equator" project to ? , It maps to a point which is twice the radius on the plane

write optimization as a function of $f(x, y)$and $g(x, y)$where $(f, g)$are obtained by stereographic projection.