# CS 217 - Light & Geometry in Computer Vision (Prof. Deva Ramanan)

Lecture 13 - Wednesday,April 29, 2009 (Scribe :Uddipan Mukherjee)

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## 1 Shape from Shading

E(p(x,y),q(x,y)) = (Image Error) + (Smoothness Penalty)+ (Non Integrability penalty) { with a known light Source }

→At Occluding contours, we can find N(x,y) and hence p(x,y) and q(x,y). But these quantities are infinite

→So we reparametrized a gradient map {p(x,y) , q(x,y)} into {f(x,y) , g(x,y)} through stereographic projection

We can also define the Energy E as E(N(x,y)$_a$, N(x,y)$_b$,N(x,y)$_c$) with a constraint that the norm has to be 1

### 1.1 Shape from unknown Light Source (point source /Lambertian Surface)

We know that $I(x, y)= \rho(x, y)N(x, y)^T.S = g(x, y)^T.S = \{g(x, y)^T G\}.\{G^{-1}S\}$

Where G is an invertible 3x3 matrix

So there exists a number of possible light sources yielding the exact same intensity.

i.e , there exists a subspace of albedo scaled normals and light sources that yield the same image

The transformed surface has both different shape (N) and Paint ($\rho$)

However , the set of shadowed points will change.

It turns out for G = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & \nu & \lambda \end{bmatrix}$ where $\lambda > 0$, the set of shadowed points remain the same

(Bas Relief Ambiguity)

# 2 Global Shading Models (MA : Ch 5)

## 2.1 Local Shading Models

This Model assumes surfaces are only illuminated by the point light sources.

We know that -

Radiosity (at P) = total radiance leaving the point P

e.g For a diffused surface and faraway point source

B(P) = $\int_{2\pi} L_o(P, \theta, \phi) cos\theta \partial\omega = \pi L_0(P)$

and

$L_o(P)$=$\rho_d(P) N(P)^T.S$

so, $B_s(P)$=$\pi\rho_d(P) N(P)^T.S$

For all sources , S visible from point P ,

B(P) = $\sum B_S(P)$

## 2.2 Global Shading Models

This model assumes surfaces are also illuminated by inter reflections (i.e. light reflected from other surfaces),e.g. corners of a room are brighter than the center of the walls.

B(p) = E(p) + $B_{ref}$(p)

where E(p) is exitance (radiance from an internal source).

From the point of view of P all other surfaces are light sources.

$B_{ref}(p) = \pi\rho_d(p) \int_{2\pi} L_i(p, \theta, \phi) cos\theta \partial\omega$

Since Radiance is constant and it travels in staright lines , we can sum Radiance leaving all surfaces headed towards P

$B_{ref}(p) = \pi\rho_d(p) \int_{world} visible(p, q) L_o(q, q \to p) cos\theta_p \frac{cos\theta_q \partial A_q}{||p-q||^2}$

where $L_o(q, q \to p)$= B(q)/$\pi$ ,and

visible(p,q) = 1 if p and q see each other and 0 otherwise

so

$B(p) = E(p) + \rho_d(p) \int_{world} visible(p, q)\ K(p, q)\ B(q)\ \partial A_q$

The term visible(p,q)K(p,q) is usually referred to as the interreflection kernel.

### 2.2.1 Solving for Radiosity

The terms E(p) and visible(p,q)K(p,q) are known and B(q) is unknown. In order to solve for B(p) we discretize the world nto small flat surface patches of constant radiosity.Then we construct a vector **B** , which contains the value of radiosity for each patch. In particular the $i^{th}$ component of **B** is the radiosity of the $i^{th}$ patch.

We write the incoming radiosity at the $i^{th}$patch due to the radiosity on the $j^{th}$patch as

$B_{j\to i} = \rho_d(p) visible(p, q) K(p, q) B_j$

So

$B_i$= $E_i$+ $\sum_j B_{j\to i}$

i.e $B_i = E_i + \sum_j K_{ij} B_j$

where $K_{ij}$ are the form factors.

This is a huge linear system which can be solved typically using Gauss Siedel Method.