CS 217: Light and Geometry in Vision **Spring 2009**

Lecture 18 — May 11

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Note: These lecture notes are still rough, and have only have been mildly proofread.

18.1 Two View Calibrated Geometry

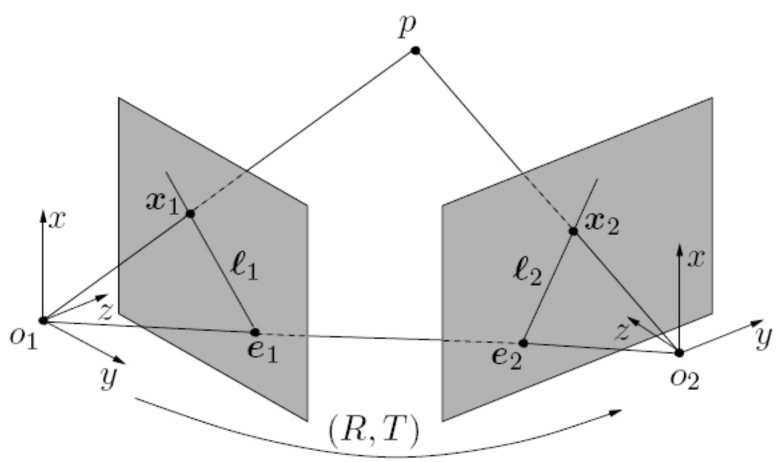

Figure 18.1. Two calibrated cameras viewing at the same point in 3D space.

- Assumption:
  (1) $K_1 = K_2 = I$.
  In general, $\lambda \mathbf{x}_1' = \begin{bmatrix} f, 0, 0 \\ 0, f, 0 \\ 0, 0, 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$,
  $x_1 = \begin{bmatrix} 1/f, 0, 0 \\ 0, 1/f, 0 \\ 0, 0, 1 \end{bmatrix} \mathbf{x}_1'$,
  $\lambda \mathbf{x}_1 = \mathbf{X}_1$.
  (2) $\mathbf{X}_2 = R\mathbf{X}_1 + T$.
  Since $\mathbf{X}_1 = \lambda_1 \mathbf{x}_1$, $\mathbf{X}_2 = \lambda_2 \mathbf{x}_2$, we have $\lambda_2 \mathbf{X}_2 = R\lambda_1 \mathbf{x}_1 + T$.

  First cross product with T:
  Recall $T \times V$, noted as $[T]_\times V$(M.V.) or $\hat{T}$(3D),

$\lambda_2 \hat{T} \mathbf{x}_2 = \hat{T} R \lambda_1 \mathbf{x}_1 + \hat{T} T$, where $\hat{T} T = \mathbf{0}$.

Then dot product with $\mathbf{x}_2$:$\lambda_2 {\mathbf{x}_2}^T \hat{T} \mathbf{x}_2 = {\mathbf{x}_2}^T \hat{T} R \lambda_1 \mathbf{x}_1$, since $\hat{T}\mathbf{x}_2$ is orthogonal to $\mathbf{x}_2$, the left hand side is $\mathbf{0}$. Thus, we have:

$$0 = {\mathbf{x}_2}^T \hat{T} R \mathbf{x}_1 \tag{18.1}$$

Equation 18.1 is the epipolar constraint for calibrated cameras. Set $E = \hat{T} R$, where E is essential matrix, we get:

$$ {\mathbf{x}_2}^T E \mathbf{x}_1 = 0 \tag{18.2}$$

- Fix $\mathbf{x}_1$, what are the set of $\mathbf{x}_2$ that satisfy epipolar constraint?
  $\mathbf{l}_2 = E\mathbf{x}_1 \Rightarrow {\mathbf{x}_2}^T \mathbf{l}_2 = 0$.

- Fix $\mathbf{x}_2$, what are the set of $\mathbf{x}_1$ that satisfy epipolar constraint?
  $\mathbf{l}_1 = E^T \mathbf{x}_2 \Rightarrow {\mathbf{l}_1}^T \mathbf{x}_1 = 0$.

- Properties of E:
  (1) $\begin{matrix} {\mathbf{e}_2}^T E \mathbf{x}_1 = 0, \forall \mathbf{x}_1 \\ {\mathbf{x}_2}^T E \mathbf{e}_1 = 0, \forall \mathbf{x}_2 \end{matrix} \Rightarrow E\mathbf{e}_1 = \mathbf{0}, {\mathbf{e}_2}^T E = \mathbf{0}^T$
  Rank(E) = 2.
  (2) How many degree of freedom are in E?
  ${\mathbf{x}_2}^T (\alpha E) \mathbf{x}_1 = 0$ implies E is only determined up to scale? Eight degree of freedom? The actuall answer is five.

18.2 Estimating E

- Eight-point algorithm due to Lonet-Higgens:
  $E = \begin{bmatrix} e_{11}, e_{12}, e_{13} \\ e_{21}, e_{22}, e_{23} \\ e_{31}, e_{32}, e_{33} \end{bmatrix}$, ${\mathbf{x}_2}^T E \mathbf{x}_1 = 0$, using $E^S = E(:)$ in Matlab.

- Definition: Kronocker product of two vectors $\mathbf{x}_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix} {x_1}' \\ {x_2}' \\ {x_3}' \end{bmatrix}$ as $\mathbf{a} =$
  $\mathbf{x}_1 \otimes \mathbf{x}_2 = \left(\mathbf{x}_2 {\mathbf{x}_1}^T\right)^S = \begin{bmatrix} x_1 \mathbf{x}_2 \\ x_2 \mathbf{x}_2 \\ x_3 \mathbf{x}_2 \end{bmatrix}$.

- Rewrite ${\mathbf{x}_2}^T E \mathbf{x}_1 = \mathbf{a}^T E^S = 0$, which is equivalent to trace representation:
  $tr\left({\mathbf{x}_2}^T E \mathbf{x}_1\right) = tr\left(\mathbf{x}_1 {\mathbf{x}_2}^T E\right) = \left(\mathbf{x}_1 {\mathbf{x}_2}^T\right)^{TS} E^{TS}$

- Assume we have n-point correspondences $\left(\mathbf{x_1}^j, \mathbf{x_2}^j\right), j = 1, 2, 3, \cdots, n$,
  $\mathbf{a}^j = \mathbf{x_1}^j \otimes \mathbf{x_2}^j$,
  $$X = \begin{bmatrix} \left(\mathbf{a}^1\right)^T \\ \left(\mathbf{a}^2\right)^T \\ \vdots \\ \left(\mathbf{a}^n\right)^T \end{bmatrix}, \; XE^S = \mathbf{0}$$

- How to solve it?
  $\overset{\min}{E} \; \|XE\|^2$, such that $\|E\| = 1 \Rightarrow$
  $\overset{\min}{E} \; E^T\left(X^TX\right)E$, such that $\|E\| = 1$.
  $E^S =$ minimum eigenvector of X^TX.

- For ideal situation with no noise, E^S has associated eigenvalue of 0.

- What happens if there are multiple eigenvalues $= 0$?
  They will be degenerate cases that occur when all points lie one a plane in 3D.