Lecture 22 — May 18

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Note: These lecture notes are still rough, and have only have been mildly proofread.

22.1 Calibrated Two-View Geometry

Let $\underset{\sim}{x_1}$, $\underset{\sim}{x_2} \in R^3$ be the homogeneous coordinates of the projection of the same point p in the two image planes, and X_1, $X_2 \in R^3$ be the 3-D coordinates of a point p relative to the two camera frames, we have

$$\begin{aligned} X_2 &= RX_1 + T \\ \lambda_1 \underset{\sim}{x_1} &= X_1 \\ \lambda_2 \underset{\sim}{x_2} &= X_2 \\ \Rightarrow \lambda_2 \underset{\sim}{x_2} &= R\lambda_1 \underset{\sim}{x_1} + T \\ \Rightarrow \lambda_2 \widehat{T} \underset{\sim}{x_2} &= \widehat{T} R \lambda_1 \underset{\sim}{x_1} \end{aligned}$$

($\widehat{\ }$ denotes cross product)

Hence epipolar constraint equation

$$\underset{\sim}{x_2}^T E \underset{\sim}{x_1} = 0$$

where essential matrix

$$E = \widehat{T} R$$

22.2 Uncalibrated Two-View Geometry

$$\begin{aligned} \lambda_1 \underset{\sim}{x_1} &= KX_1 \\ \Rightarrow \lambda_1 K^{-1} \underset{\sim}{x_1} &= X_1 \\ \Rightarrow \lambda_2 K_2^{-1} \underset{\sim}{x_2} &= R\lambda K_1^{-1} \underset{\sim}{x_1} + T \end{aligned}$$

Recall

$$K = \begin{bmatrix} f_{sx} & f_{s0} & 0_x \\ 0 & f_{sy} & 0_y \\ 0 & 0 & 1 \end{bmatrix}$$

Hence epipolar constraint equation becomes

$$\underset{\sim}{x_2}^T F \underset{\sim}{x_1} = 0$$

where fundamental matrix

$$F = K_2^{-T} \widehat{T} R K_1^{-1}$$

Estimating F
All fundamental matrices can be written as

$$F = U \Sigma V^T$$

where

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \sigma_1 > 0, \sigma_2 > 0$$

Any 3×3 matrix of rank 2 is a valid fundamental matrix

$$XF^s = 0$$

F^s denotes stacked version of the matrix F
For example, define

$$F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \in R^{3\times3}$$

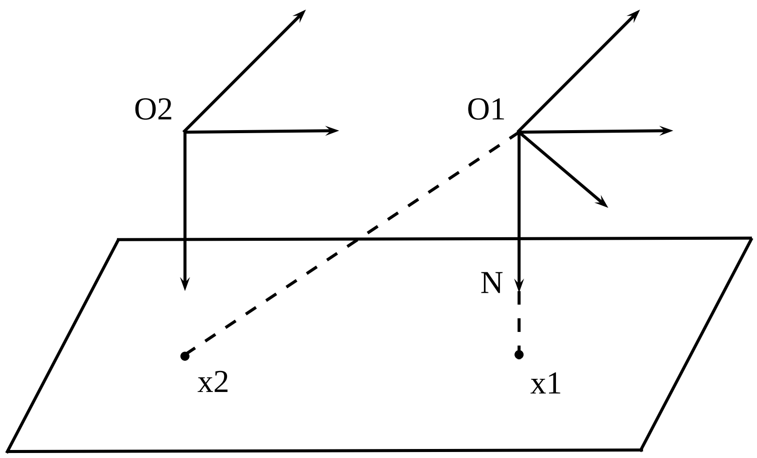
then

$$F_s = [f_{11}, f_{21}, f_{31}, f_{12}, f_{22}, f_{32}, f_{13}, f_{23}, f_{33}] \in R^9$$

22.3 Homographies

22.3.1 Scene is planner

What happens in two-view geometry(calibrated & uncalibrated) when scene is planner?



$$
\begin{aligned}
X_2 &= RX_1 + T \\
\widehat{N}^T \underset{\sim}{x}_1 &= n_1x_1 + n_2x_2 + n_3x_3 = d \\
where\ \widehat{N} &= \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}, such\ that\ \Sigma \widehat{N_i}^2 = 1 \\
\Rightarrow \frac{1}{d}\widehat{N}^T \underset{\sim}{x}_1 &= 1 \\
\Rightarrow \underset{\sim}{x_2} &= R\underset{\sim}{x_1} + \frac{1}{d}T\widehat{N}^T \underset{\sim}{x}_1 \\
&= (R + \frac{1}{d}T\widehat{N}^T)\underset{\sim}{x}_1 \\
\Rightarrow X_2 &= HX_1,\ where\ H = [R + \frac{1}{d}T\widehat{N}^T]
\end{aligned}
$$

2D points are related by

1) $\lambda_2 \underset{\sim}{x_2} = H\lambda_1\underset{\sim}{x_1},\ calibrated,\ where\ H = [R + \frac{1}{d}T\widehat{N}^T]$

2) $\lambda_2 \underset{\sim}{x_2} = H\lambda_1\underset{\sim}{x_1},\ uncalibrated,\ where\ H = K^{-T}(R + \frac{1}{d}T\widehat{N}^T)K^{-1}$

How does this relate to E/F?

$$
\underset{\sim}{x_2}^T E \underset{\sim}{x_1} = 0,\ maps\ points\ to\ lines
$$
$$
\underset{\sim}{x_2} \sim H\underset{\sim}{x_1},\ maps\ points\ to\ points
$$

22.3.2 Rotating camera

$$\underset{\sim}{x_2} = R\underset{\sim}{x_1}$$

We can use homographies to construct mosaics(collection of images from rotating camera) or to rectify images for stereo matching.

Estimate E from point correspondence
Select any $u \in R^3$

$$\begin{aligned} \underset{\sim}{x_2} &\sim H\underset{\sim}{x_1} \\ \Rightarrow (\widehat{u}\underset{\sim}{x_2})^T H\underset{\sim}{x_1} &= 0 \\ \Rightarrow \underset{\sim}{x_2}^T \widehat{u} H\underset{\sim}{x_1} &= 0 \\ E &= \widehat{u}H, \ for\ any\ u\ \in R^3\ (no\ unique\ E) \end{aligned}$$

Estimate H

$$\begin{aligned} \underset{\sim}{x_2} &\sim H\underset{\sim}{x_1} \\ \Leftrightarrow \lambda_2 \underset{\sim}{x_2} &= H\lambda_1\underset{\sim}{x_1} \\ \Leftrightarrow \widehat{\underset{\sim}{x_2}} H\underset{\sim}{x_1} &= 0 \\ \Rightarrow a^T H^s &= 0 \end{aligned}$$

where

$$a = \underset{\sim}{x_1} \bigotimes \underset{\sim}{x_2}, a \in R^{9\times 3}$$

Construct

$$\text{X=} \begin{bmatrix} {a_1}^T \\ \vdots \\ {a_n}^T \end{bmatrix}_{3n\times 9}$$

such that

$$XH^s = 0$$

Solve H^s with SVD.