CS 217: Light and Geometry in Vision	Spring 2009

Lecture 4 — April 13

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Note: These lecture notes are still rough, and have only have been mildly proofread.

4.1 BRDF for Different Types of Surfaces

BRDF(Bidirectional reflectance distribution function) can be defined as the ratio of the radiance in the outgoing direction to the incident irradiance:

$$f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{L_e(x, \theta_e, \phi_e)}{L_i(x, \theta_i, \phi_i)\cos\theta_i \delta\omega} = \frac{L(\theta_e, \phi_e)}{E(\theta_i, \phi_i)} \tag{4.1}$$

It describes how bright a surface appears when viewed from one direction while light falls on it from another.

4.1.1 Lambertian Surface

• *Lambertian Surface*: the surface which appears equally bright from all viewing directions and reflects all incident light. The BRDF of Lambertian surface is:

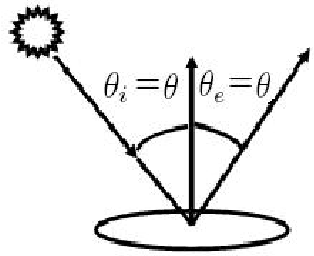
$$f = \rho_{brdf} = \frac{1}{\pi}\rho_d \tag{4.2}$$

where ρ_d is a Directional Hemisphere Reflectance which is independent of direction we also call it "Albedo":

$$\rho_d = \int_{2\pi} f(\theta_e, \phi_e, \theta_i, \phi_i)\cos\theta_e d\omega_e$$

4.1.2 Ideal Specular Surface

• *Ideal Specular Surface*: the surface that reflects all the light coming from the direction (θ_i, ϕ_i) into the direction $(\theta_i, \phi_i + \pi)$.



In this case the BRDF of the surface is proportional to the product of two impulses $\delta(\theta_i - \theta_e)$ and $\delta(\phi_i + \pi - \phi_e)$:

$$f(\theta_i, \phi_i, \theta_e, \phi_e) = k\delta(\theta_i - \theta_e)\delta(\phi_i + \pi - \phi_e) \tag{4.3}$$

where

$$k = \frac{1}{\sin\theta_i \cos\theta_i} \tag{4.4}$$

and in this case General form of BRDF is:

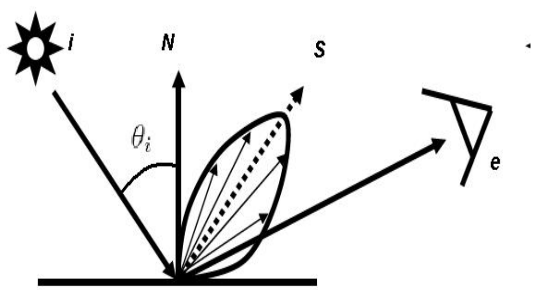
$$f(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{k\delta(\theta_i - \theta_e)\delta(\phi_i + \pi - \phi_e)}{\sin\theta_i \cos\theta_i} \tag{4.5}$$

So now if we want to calculate the Radiance we will get:

$$L(\theta_e, \phi_e) = \int_{2\pi} \frac{k\delta(\theta_i - \theta_e)\delta(\phi_i + \pi - \phi_e)}{\sin\theta_i \cos\theta_i} E(\theta_i, \phi_i) \cos\theta_i d\omega_i \tag{4.6}$$

4.1.3 Phong Model

Oftentimes incoming radiance gets reflected out in a lobe of directions



- Phong BRDF: $f(\theta_i, \phi_i, \theta_e, \phi_e) = f(\hat{i}, \hat{s}) \propto \rho_s(\hat{s} \cdot \hat{e})^n$ where $\hat{s}$ is unit vector given by $(\theta_i, \phi_i + \pi)$.

  n- control width of specular lobe (this is not possible physically)

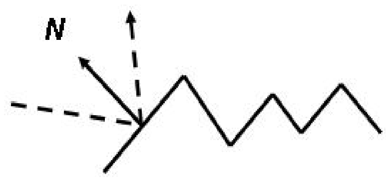
4.1.4 Diffuse + Lambertian Model

Common Computer Graphics model for both Diffuse and Specular components of a surface is:

$$L(x, \theta_e, \phi_e) = \rho_d(x) \int_{2\pi} L_i(x, \theta_i, \phi_i) \cos\theta_i d\omega_i + \rho_s(x)(\hat{s} \cdot \hat{e})^n \tag{4.7}$$

4.1.5 Torrent Sparrow BRDF

• From physics: assumes surface is constructed of micro-facets with mirror reflectance. BRDF has a parameter that specifies the distribution of micro-facets normals.



4.2 Common source + BRDF arrangements

1. Lambertian object with a point source which is far-away

   Define: Exitance =Radiousity of a point light source

$$E = \int_{2\pi} L_e(x, \theta_e, \phi_e) \cos\theta_e d\omega_e \text{ where x-is a point light} \tag{4.8}$$

   Now assume this point light source is at (θ_s, ϕ_s)

$$L_i(x, \theta_i, \theta_i) = \frac{E\delta(\theta_i - \theta_s)\delta(\phi_i - \phi_s)}{\sin\theta_s} \tag{4.9}$$

   Knowing the BRDF for Lambertian surface we can show:

$$L_e(x, \theta_e, \theta_e) = \frac{1}{\pi}\rho_d E \cos\theta_i \tag{4.10}$$

2. What does Lambertian object look like at constant illumination E

$$L_i(x, \theta_i, \theta_i) = E \tag{4.11}$$