CS 217: Light and Geometry in Vision **Spring 2009**

Lecture 9 — April 17

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Note: These lecture notes are still rough, and have only have been mildly proofread.

This is the danger environment.

Agenda:

- Photometric Stereo.

9.1 Review

Assumptions: Single far spherical source with lambertian surface under orthographic projection.

$$I(x,y) = L_e(x,y) = g(x,y)^T S$$

where $g(x,y) = p(x,y)N(x,y)$. Note that S is known and g(x,y) is unknown.

We can solve for g(x,y) by solving for the least squares solution for each pixel independently. To do so, we define:

$$V = \begin{pmatrix} S_1^T \\ ... \\ S_N T \end{pmatrix}$$

$$C(x,y) = \begin{pmatrix} I_1(x,y) \\ ... \\ I_N(x,y) \end{pmatrix}$$

and let:

$$g = (V^T V)^{-1} V_C$$

Now we have both N(x,y) and P(x,y) using the equations in the previous lecture note.

We can visualize the normal field as a ”needle diagram.” A ”needle diagram” has the following property: for every pixel, there is a ”needle” pointing along the normal.

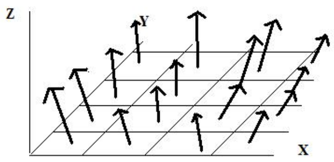

Figure 9.1. needle diagram

9.2 Surface Reconstruction

The first step to surface reconstruction is to parameterize the 3D surface as a height feild: $(x, y, f(x, y))$, where f(x,y) is the corresponding height to point (x,y).

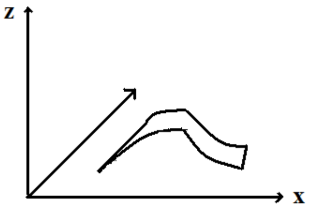

Figure 9.2. Surface reconstruction example

In doing so, we will perform a taylor expansion on f(x,y) to get:

$$f(x + \triangle x, y + \triangle y) = f(x, y) + \frac{df}{dx}\triangle x + \frac{df}{dy}\triangle y + Higher.Order.Terms. \tag{9.1}$$

For convenience, we will define: p = $\frac{df}{dx}$ and q = $\frac{df}{dy}$

Then, we will look at what happens when we take a small $\triangle x$ step ($\triangle x = 1$) and a small $\triangle y$ step ($\triangle y = 1$) at point (x,y). We can visualize this as:

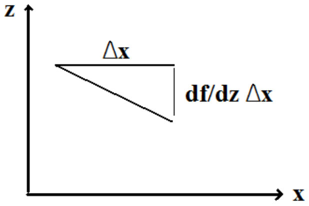

Figure 9.3. Visualization of taking a smal step in the x-direction. The same logic can be applied for the y-direction.

Formally, we can define:

$$< 1, 0, p >= r_x \tag{9.2}$$

$$< 0, 1, p >= r_y \tag{9.3}$$

$$n = \frac{r_x \times r_y}{\|r_x \times ry\|} = \frac{< -p, -q, -1 >}{\sqrt{p^2 + q^2 + 1}} \tag{9.4}$$

If we were given a normal, $n = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

We can define p and q as:

$$p = \frac{-a}{c} \tag{9.5}$$

$$q = \frac{-b}{c} \tag{9.6}$$

$$\tag{9.7}$$

Goal:

- At this point, we essentially want to find f(x,y) such that: $\frac{df}{dx} = p$ and $\frac{df}{dy} = q$

9.3 Digression into reflecance maps

We will now take a step back and look at a different approach. Instead of working with normals and needle diagrams as the representation, we could have chosen the gradient (p,q). This is also known as a reflectance map.

For this, we will assume unit albedos and unit light source:

$$I(x, y) = \rho_d(x, y)N(x, y)^T S(x, y) \approx \hat{N}(x, y)^T \hat{S}(x, y) \tag{9.8}$$

$$R(p, q) = (\frac{< -p, -q, 1 >}{\sqrt{p^2 + q^2 + 1}})(\frac{< -p_S, -q_S, 1 >}{\sqrt{p^2 + q^2 + 1}}) \tag{9.9}$$

$$\tag{9.10}$$

A Reflectance map expresses brightness as a function of surface orientation (pq) for a fixes illumination condition.

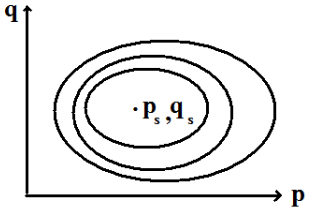

Figure 9.4. Reflectance map

Equivalently, we can think of a unit-radius sphere (Gaussian Sphere) where its surface orientation is mapped to its intensity. That is to say, the surface intensities of the sphere correspond to the orientation of the ligth source. This is a practical trick for determining light source location. We can visually think of the p,q map as:

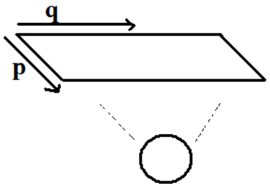

Figure 9.5. Gradient space

Note that the (p,q) gradient space is a projection of the viewable hemisphere of a gaussian sphere.

9.4 Surface reconstruction (continued)

Recall that we want to find f(x,y) such that $\frac{df}{dx} = p$ and $\frac{df}{dy} = q$

To obtain an intuitive understanding of the problem, we will first look at reconstruction in 1D (e.g., want to find f(x) such that: $\frac{df(x)}{dx} = p$). This will yield:

$$f(x) = \int_{x_0}^{x} p(x)dx + k \tag{9.11}$$

In 2D, we will obtain:

$$f(x,y) = \int_{x_0,y_0}^{x,y} pdx + qdy + f(x_0, y_0) \tag{9.12}$$

Integrability constraint: In ideal case it should not matter which path we take.

$$\frac{d^2 f}{dxdy} = \frac{d^2 f}{dydx} \tag{9.13}$$

Integrating the gradient field $\frac{df(x,y)}{dx}$ $\frac{df(x,y)}{dy}$ over any closed curve = 0.
We can use this to check the quality of p(x,y) and q(x,y).

The correct approach is to find f(x,y) such that:

$$f(x,y) = \min_{f(x,y)} \int \int (\frac{df}{dx} - p)^2 + (\frac{df}{dy} - q)^2 dxdy \tag{9.14}$$