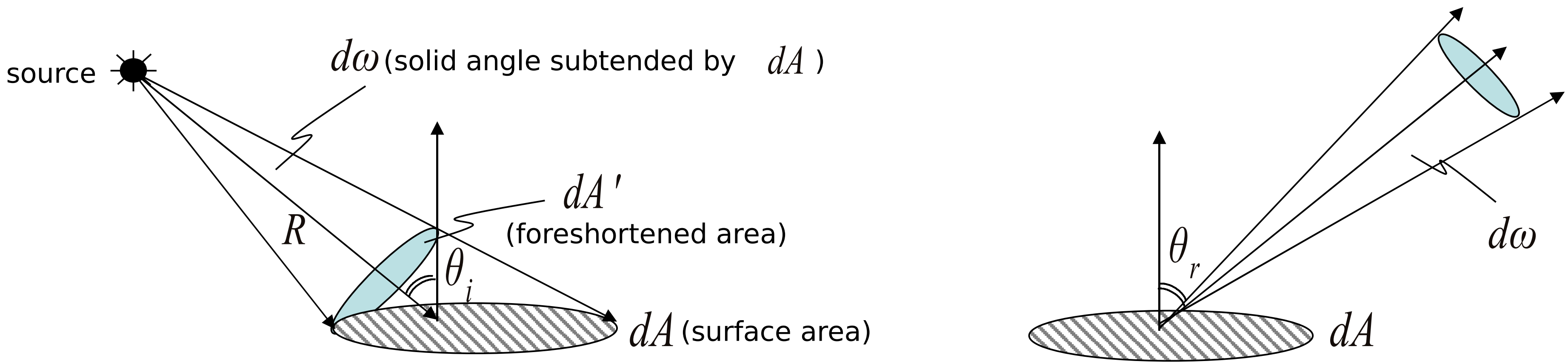
Radiometric concepts

Slide from Srinivas Narasimhan



(1) Solid Angle : $d\omega = \frac{dA'}{R^2} = \frac{dA \cos\theta_i}{R^2}$ (steradian)

Solid angle subtended by hemisphere = 2π
(half surface area of sphere)

(2) Radiant Intensity of Source : $\frac{dP}{d\omega}$ (watts/steradian)

Power emitted per unit solid angle
"Flux" synonymous with power

(3) Surface Irradiance : $E = \frac{dP}{dA}$ (watts / m*m)

Power incident per unit surface area.

(4) Surface Radiance:

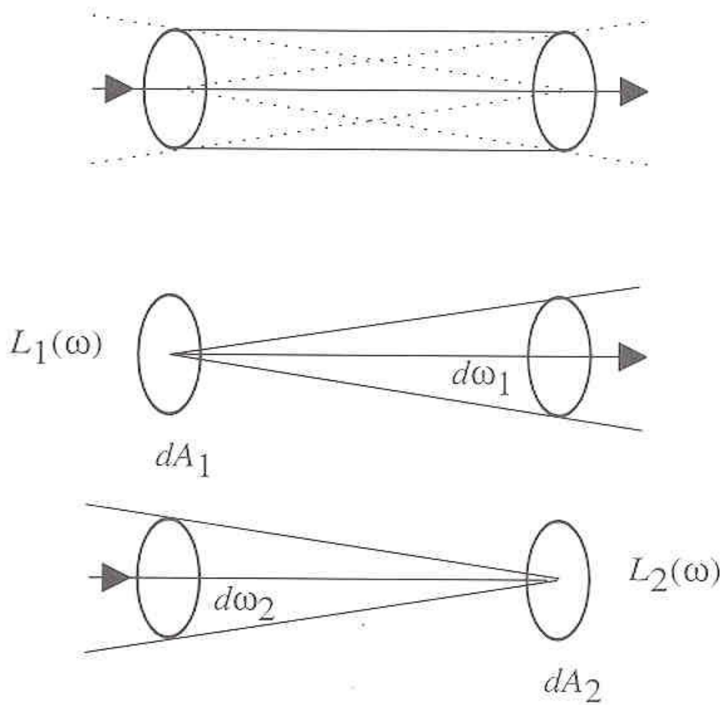
$$L = \frac{d^2 P}{(dA \cos\theta_r)\, d\omega}$$ (watts / m^2 steradian)

- Flux emitted per unit foreshortened area per unit solid angle.
- *L* depends on direction θ_r
- Surface can radiate into whole hemisphere.
- *L* depends on reflectance properties of surface.

Light Transport Assumption

- Radiance is constant as it propagates along ray from surface to camera

$$L = \frac{\partial P}{\partial \omega \, \partial A \cos\theta}$$



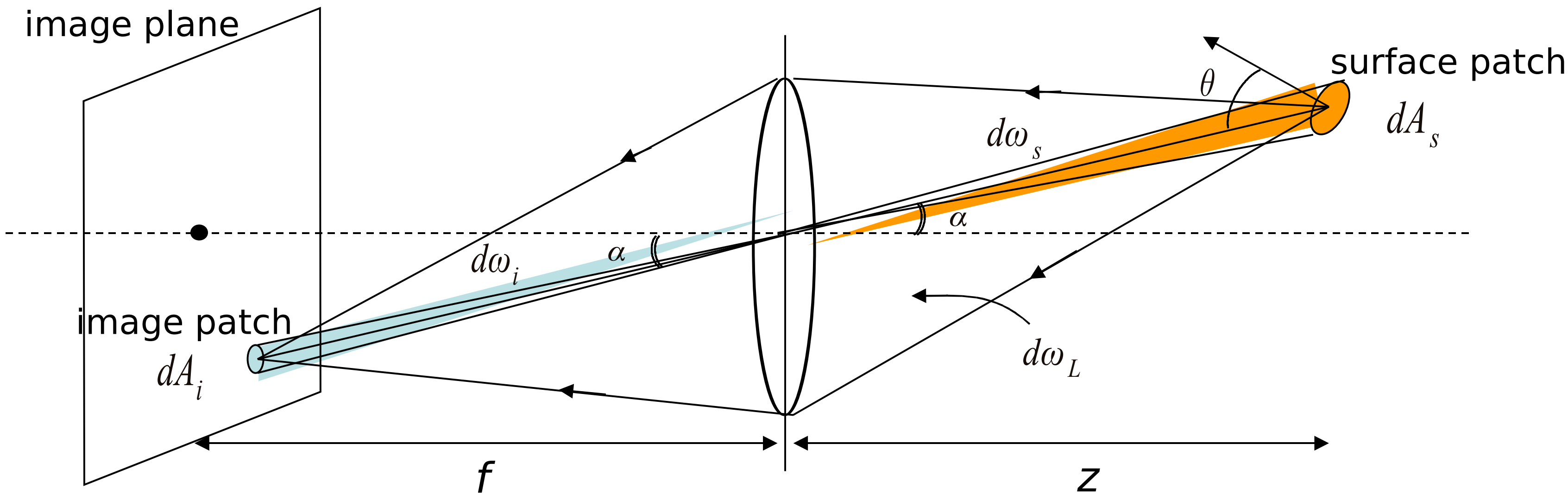
$$\partial P_1 = \partial P_2$$

$$L_1 \partial \omega_1 \partial A_1 = L_2 \partial \omega_2 \partial A_2 \quad \text{for} \quad \theta = 0$$

$$\partial \omega_1 = \frac{\partial A_2}{r^2} \qquad \partial \omega_2 = \frac{\partial A_1}{r^2}$$

$$L_1 = L_2$$

Relation between Image Irradiance E and Scene Radiance L



- Solid angles of the double cone (orange and green):

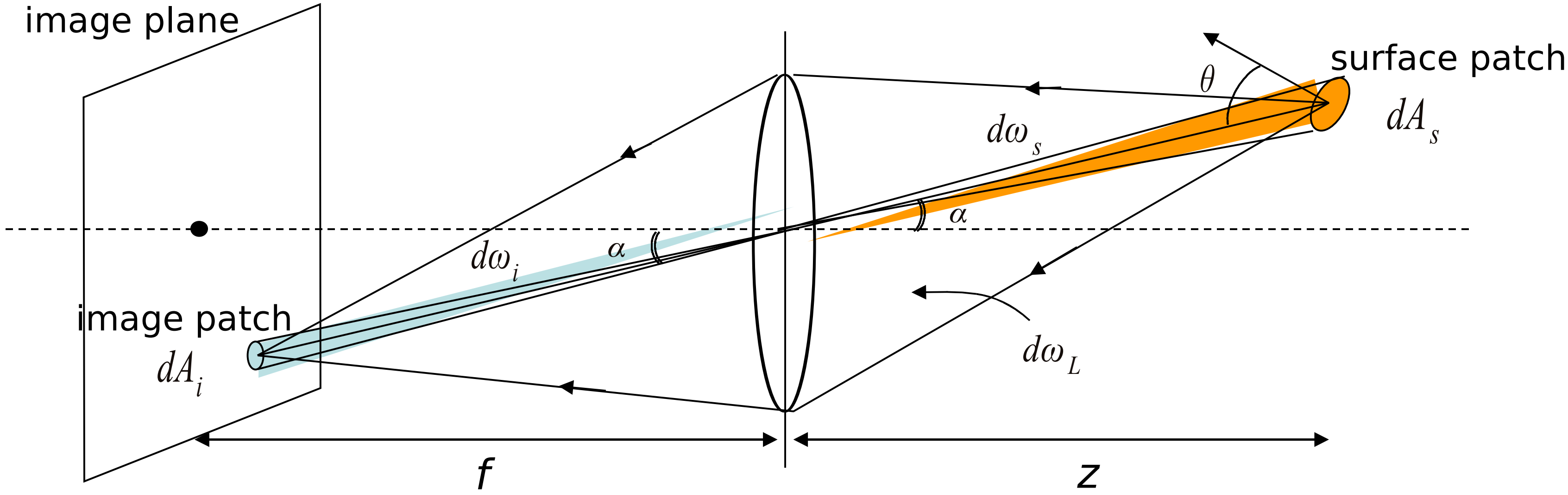
$$d\omega_i = d\omega_s \qquad \frac{dA_i \cos\alpha}{(f/\cos\alpha)^2} = \frac{dA_s \cos\theta}{(z/\cos\alpha)^2}$$

$$\frac{dA_s}{dA_i} = \frac{\cos\alpha}{\cos\theta}\left(\frac{z}{f}\right)^2 \quad \text{(1)}$$

- Solid angle subtended by lens:

$$d\omega_L = \frac{\pi d^2}{4}\frac{\cos\alpha}{(z/\cos\alpha)^2} \quad \text{(2)}$$

Relation between Image Irradiance E and Scene Radiance L



- Flux received by lens from dA_s = Flux projected onto image dA_i

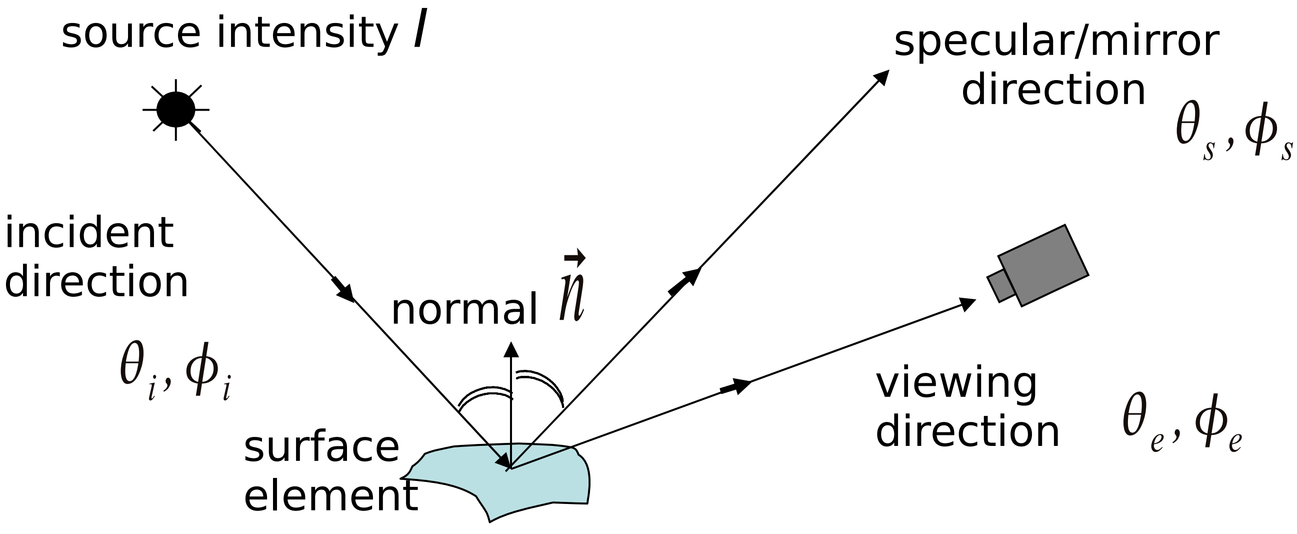
$$L \left(dA_s \cos\theta \right) d\omega_L = E \; dA_i \quad \longrightarrow \quad (3)$$

- From (1), (2), and (3):

$$E = L \, \frac{\pi}{4} \left(\frac{d}{f} \right)^2 \cos \alpha^4$$

  - Image irradiance is proportional to Scene Radiance
  - Small field of view → Effects of 4th power of cosine are small.

Specular Reflection and Mirror BRDF

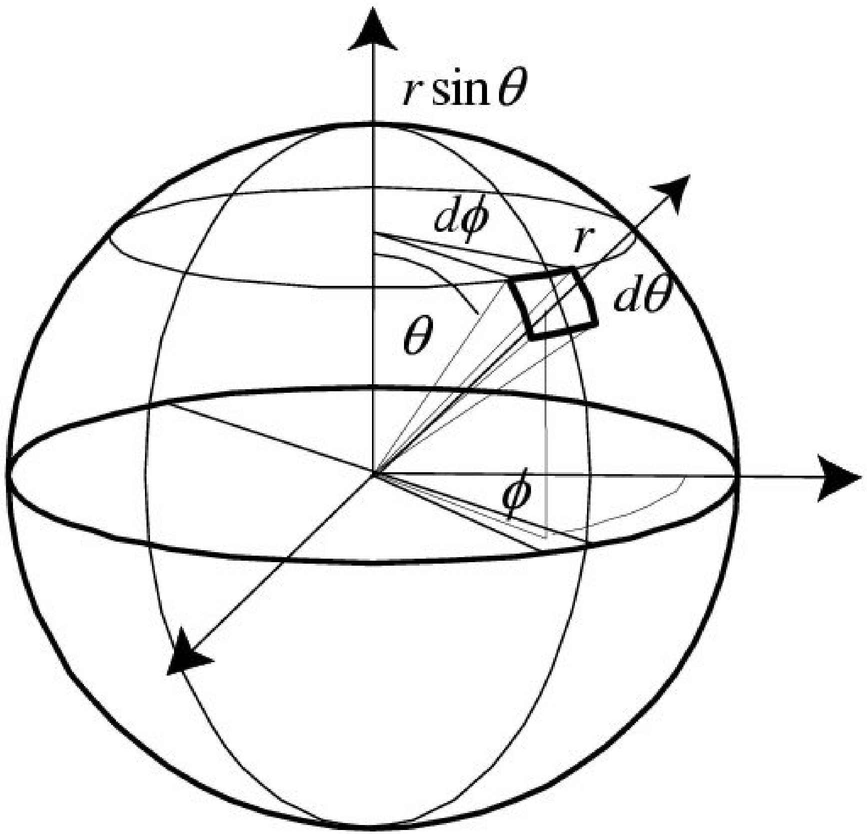


- **Very smooth surface.**
- All incident light energy reflected in a SINGLE direction. (only when $\vec{v} = \vec{r}$)
- Mirror BRDF is simply a double-delta function :

$$f(\theta_i, \phi_i; \partial\theta_i, \phi_i) = \rho_s \frac{\delta(\theta_i - \theta_e)\delta(\phi_i + \pi - \phi_e)}{\sin\theta_i \cos\theta_i}$$

- Surface Radiance : $L = I \ \rho_s \ \delta(\theta_i - \theta_v) \ \delta(\varphi_i + \pi - \varphi_v)$

Differential Solid Angles

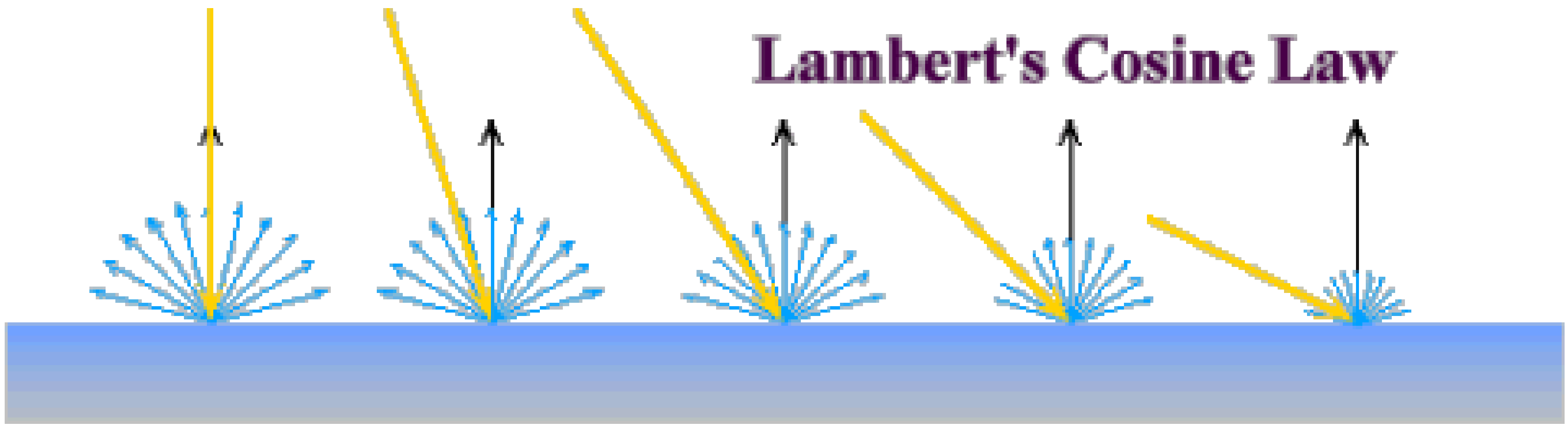


$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

Gonioreflectometers

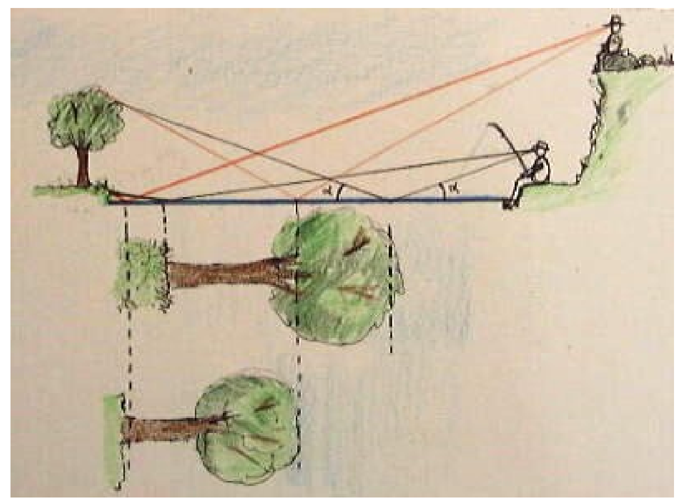

Lambert's Cosine Law

Specular Reflections in Nature

It's surprising how long
the reflections are when
viewed sitting on the river
bank.

Compare sizes of objects and their
reflections!

The reflections when seen from a lower
view
point are always longer than when
viewed
from a higher view point.



White-out Conditions from an Overcast Sky

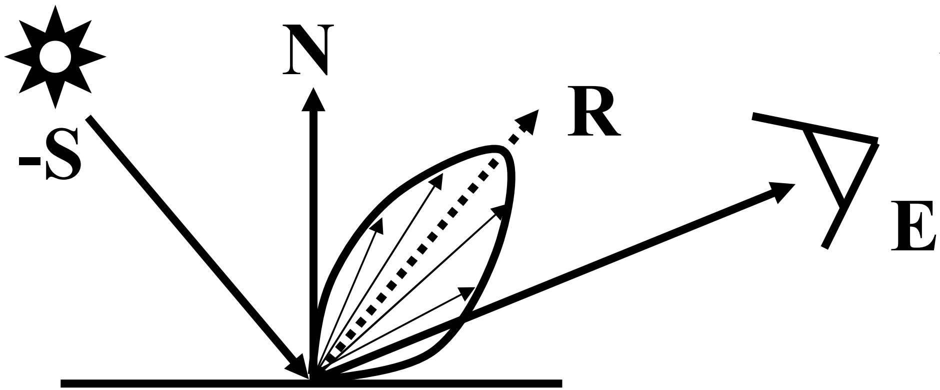
CAN’T perceive the shape of the snow covered terrain!

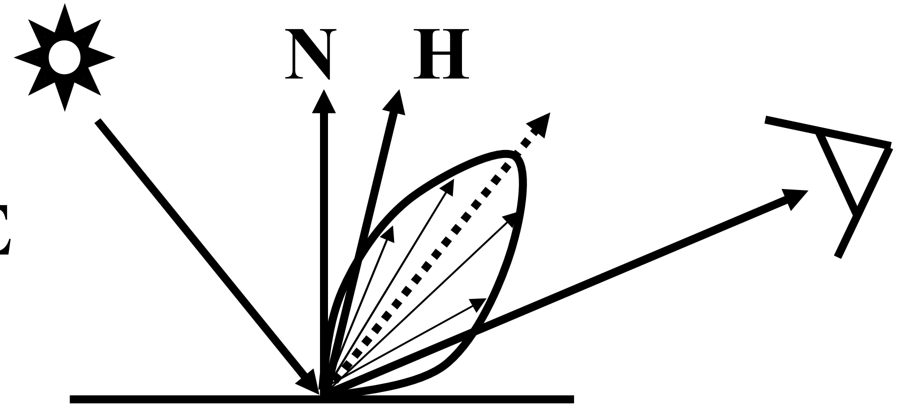
CAN perceive shape in regions
lit by the street lamp!!

WHY?

Phong Model: An Empirical Approximation

- How to model the angular falloff of highlights:





$$L = I\,\rho_s \left(R \cdot E\right)^{n_{shiney}}$$

$$R = -S + 2\left(N \cdot S\right) N$$

Phong Model

$$L = I\,\rho_s \left(N \cdot H\right)^{n_{shiney}}$$

$$H = \left(E + S\right)/2$$

Blinn-Phong Model

- Sort of works, easy to compute
- But not physically based (no energy conservation and reciprocity).
- Very commonly used in computer graphics.

Glossy Surfaces

- Delta Function too harsh a BRDF model
  (valid only for highly polished mirrors and metals).

- Many glossy surfaces show broader highlights in addition to mirror reflection.

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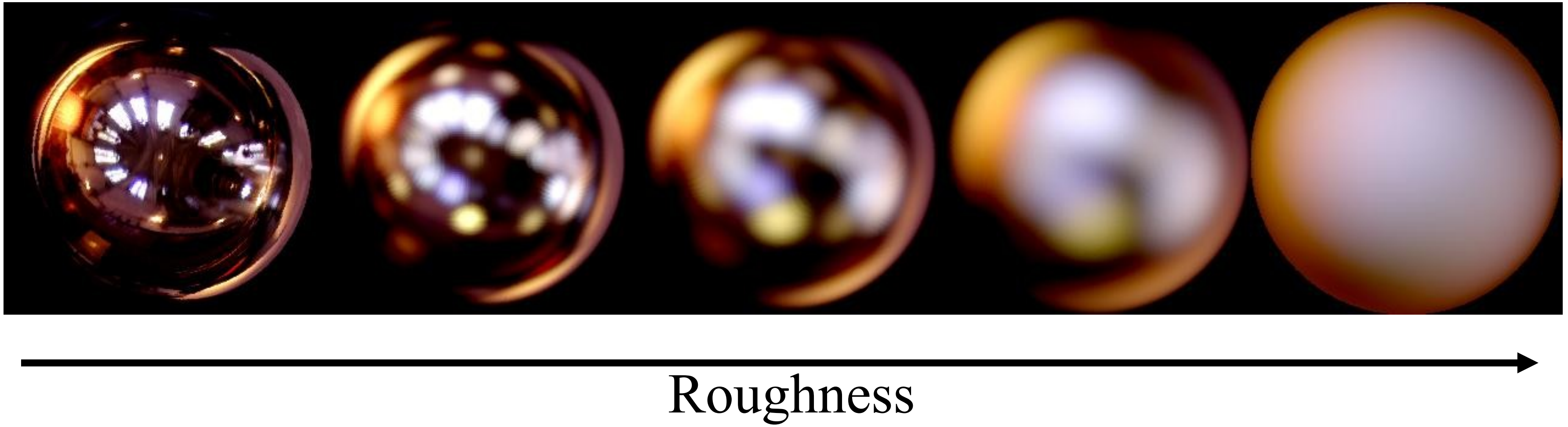
- Surfaces are not perfectly smooth – they show micro-surface geometry (roughness).

- Example Models : Phong model

  Torrance Sparrow model

Blurred Highlights and Surface Roughness



Phong Examples

- These spheres illustrate the Phong model as *lighting direction* and n_{shiny} are varied: