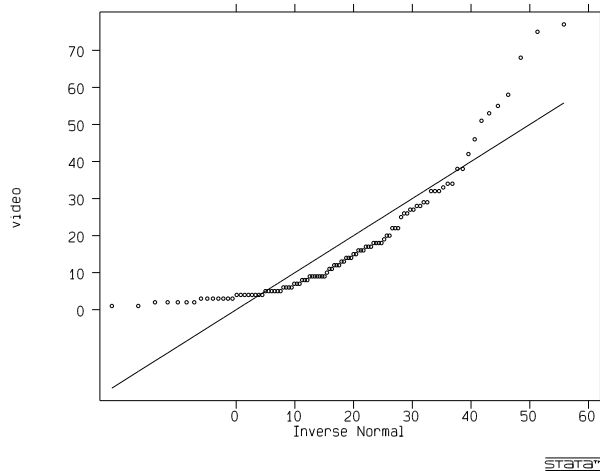


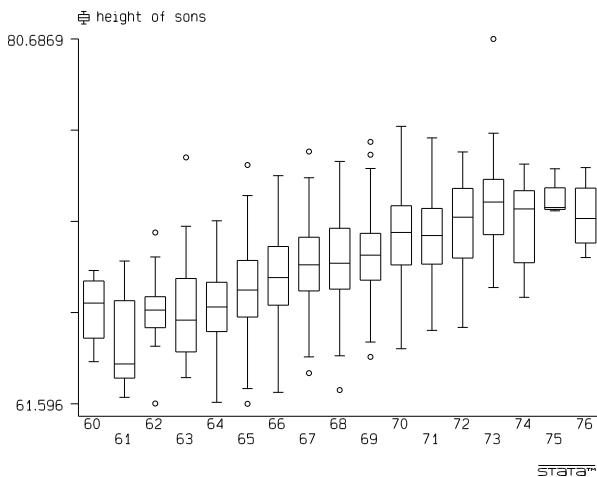
Statistics 7—Practice Midterm Examination

1. Here we consider the number of videotaped movies rented each week per city block in Cambridge. Data were collected on 100 city blocks during a particular week and a normal quantile plot appears below.
 - (a) Describe the shape of the distribution.
 - (b) What is the 95th percentile of the data?
 - (c) Suppose we use a normal distribution with the sample mean and variance to model the data. According to this model, would the 95th percentile be lower than, higher than, or the same as what you reported in part (b)? No calculations are necessary, but an explanation of your answer is required.



2. A factory owner is interested in the effect of room temperature on the performance of tasks requiring manual dexterity. She chooses temperatures $70^{\circ}F$ and $90^{\circ}F$ as treatments. The response variable is the number of correct insertions, during a 30-minute period, in an elaborate peg-and-hole apparatus that requires the use of both hands simultaneously. Each subject is trained on the apparatus and then asked to make as many insertions as possible in 30 minutes of continuous effort.
 - (a) Outline a completely randomized design to compare dexterity at 70° and 90° . Suppose that twenty subjects S_1, S_2, \dots, S_{20} are available for the experiment. Assume only one worker can be tested each day.
 - (b) Because individuals differ greatly in dexterity, the factory owner decides to conduct a special kind of randomized block design called a matched pairs design. Describe in detail the design of a matched pairs design in which each subject serves as his or her own control. (Use only the first ten subjects.)
 - (c) Which of two designs would you prefer, the completely randomized design or the matched pairs design?
 - (d) If ten sets of the apparatus were available, all ten trials at 70° could be run at the same time in one large room, and then all ten trials at 90° on a second day. Would this change the interpretation of the results? (Answer for the completely randomized experiment in part (a).)
 - (e) Plants in different parts of the country are kept at different temperatures. Instead of the study above, the factory owner collects information on manual dexterity from workers at various plants. Is this an experiment, survey, or observational study? Briefly justify your answer.

3. In this problem we will investigate the relationship between the height of fathers and their adult sons, using data from 1000 such pairs. A series of box plots is given below. All data are reported in inches.



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. summarize
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Variable	Obs	Mean	Std. Dev.	Min	Max
Father	1000	68.05679	2.671891	59.97793	76.08667
Son	1000	69.15203	2.609882	61.59598	80.6869

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.regress Son Father
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Son	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
Father		.0268319	18.090	0.000	.4327477 .5380544
_cons	36.1172	1.827496	19.763	0.000	32.53102 39.70337

- Discuss the relationship between father's and son's height based on the box plots. Describe how the conditional mean and conditional variance of son's heights change as a function of father's heights.
- Based on the output given above (note some output has been removed), give an equation for the regression line for predicting son's heights. Is this line an adequate description of the relationship between the two variables? Why or why not? Interpret the model parameters in the equation.
- Compute the correlation coefficient between father's and son's heights. What percent of the variation in son's heights is explained by father's heights?
- Suppose a father is 72 inches tall, how tall does the regression line predict his son will be? Call your answer x .
- Now consider a son who is x inches tall. (Here x is the value you computed in part (d).) Does the appropriate regression line predict his father to be shorter than, taller than, or equal to 72 inches tall. No calculations are required, but you should explain your reasoning.
- Again consider fathers who are 72 inches tall. According to the model, what is the conditional standard deviation for the heights of sons of such fathers?
- What is the probability that the son of a 72-inch-tall father is more than 74 inches tall?

4. The video rental board (a regulatory agency that supervises the video rental industry) wants to gather information on video rentals in Somerville. They randomly sample and call 240 adults and ask whether the respondent rented a movie in the last week. Assume that in the entire city 10% of adults rented a movie last week. Let X be the number in the survey who rented a movie.
- What is the distribution of X ? Explain.
 - What are the mean and variance of X ? What are the mean and variance of the proportion of people who rented a movie?
 - What is the chance that 30 or more people in the sample rented a movie? What assumptions and theory underlie your calculations?
5. Ann and Bob are playing the game Two-Finger Morra. Each player shows either one or two fingers and at the same time calls out a guess for the number of fingers the other player will show. If a player guesses correctly and the other player does not, the player wins a number of dollars equal to the total number of fingers shown by both players. If both or neither guesses correctly, no money changes hands. On each play both Ann and Bob choose one of the following options:

Choice	Show	Guess
A	1	1
B	1	2
C	2	1
D	2	2

- Give the sample Space \mathcal{S} by writing all possible choices for both players on single play of the game.
- Let X be Ann's winning on a single play. (If Ann loses \$2, then $X = -2$; when no money changes hands, $X = 0$.) Write the value of the random variable X next to each of the outcomes you listed in part (a). **[This part of the problem is tedious. You should not expect a problem like this on the exam... but parts (a, c, d, and e) are typical exam questions.]**
- Now assume Ann and Bob chose independently of each other. Moreover, they are both lay so that all four choices listed above are equally likely. Find the probability distribution of X . (I.e., list $\Pr(X = x)$ for each possible value of x .)
- What is the chance that Ann wins on any one play?
- Suppose Ann and Bob decide to play four rounds. What is the chance that money changes hands in at least one round?