ICS 162 – Fall 2003 – Midterm

Name:

Student ID:

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6:

Total:
1. (30 points)
a. (20 points) Find a deterministic finite automaton that recognizes the same languages as the one recognized by the following nondeterministic finite automaton.

```
0,1
1 1
0
```

b. (10 points) If a nondeterministic finite automaton has three states, what is the largest number of states that might be needed when it is converted to a deterministic finite automaton?
2. (20 points)
Find a regular expression describing the set of strings of zeros and ones in which every zero is followed by two consecutive ones. For instance, 1011011 is in this language, but 0011 is not.
3. (15 points)
For each of the following three grammars, state whether or not the grammar is in Chomsky Normal Form. If it is not, explain why not.

\[
\begin{align*}
S & \rightarrow \epsilon \mid XY \\
X & \rightarrow 0 \mid YS \\
Y & \rightarrow 1 \mid XX
\end{align*}
\]

\[
\begin{align*}
S & \rightarrow AB \mid AA \mid BB \\
A & \rightarrow 0 \mid BA \\
B & \rightarrow 1
\end{align*}
\]

\[
\begin{align*}
S & \rightarrow 0P \mid 1Q \mid PQ \\
P & \rightarrow 1 \mid QP \\
Q & \rightarrow 0 \mid PQ
\end{align*}
\]
4. (15 points)
Show that the following grammar is ambiguous:

\[ S \rightarrow 0S1 \mid 01S \mid \epsilon \]
5. (20 points)

Let $L = \{ww \mid w \in 0^*1^*\}$. That is, $L$ consists of strings of zeros and ones in which the first and second halves of the string are equal to each other, and each half contains a single one and any number of zeros. Use the pumping lemma to show that $L$ is not regular.
6. (Extra credit: 20 points)

Let \( L = \{ww | w \in 0^*10^*\} \) be the same language as described in problem 5. Find a context free grammar for \( L \). You do not need to use Chomsky Normal Form for your grammar.

*Hint: the expression \( ww \) shows how to split strings in \( L \) into two equal parts. Instead, split the strings into two parts that might not be equal.*
You may use this page (or the back of the other pages) as scratch paper.