CS 163 – Fall 2022 – Final exam

Name:

UCInet id:

This is a closed book, closed note exam. Calculators or other electronic devices are not allowed.

Do not open this exam until told to start by the instructor.

Please write your answers ONLY on the front side of each page.
Answers written elsewhere will not be scanned and will not graded.

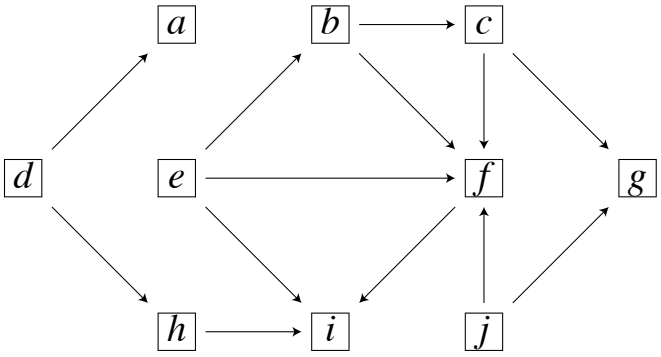
You may use the back sides of the pages as scratch paper.
Do not unstaple the pages.

1. (15 points) Draw a complete graph with four vertices, and find six different weights for the six edges of your graph, with the property that the minimum spanning tree and the maximum spanning tree share an edge. Show both trees. (Hint: at least one of these two trees should not be a path.)

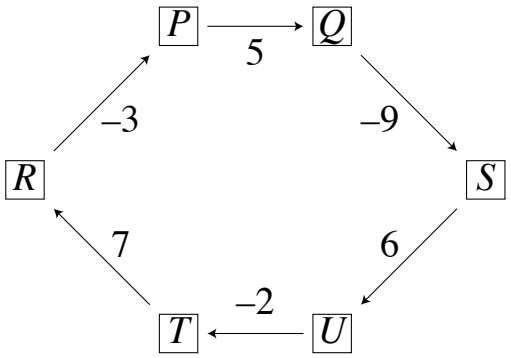
2. (15 points) For the directed acyclic graph shown below:

   (a) List all of the vertices that could be the first vertex in a topological order.

   (b) List all of the vertices that could be the last vertex in a topological order.



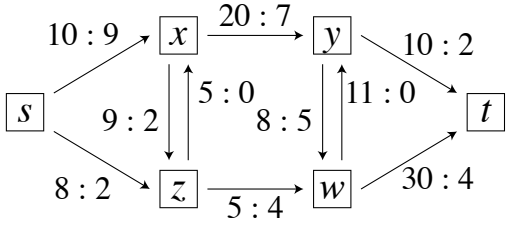
3. (15 points) If Johnson's algorithm is applied to find shortest paths between all pairs of vertices in the graph shown below, one of the steps of the algorithm will change the edge lengths of the graph to make them all ≥ 0. Calculate and show the new edge lengths that it uses. (Hint: You should be able to skip the step of the algorithm that uses Bellman–Ford, and instead calculate distances from the new vertex s directly.)



4. (15 points) State the running time of the exponential-time dynamic programming algorithm for the traveling salesperson problem, using O-notation, as a function of the number of vertices, n.

5. (15 points) Let G be a graph with n vertices and degeneracy two. What is the largest possible number of edges that G could have? (Your answer should be a formula involving n. Do not use O-notation. Hint: work backwards from smaller graphs to larger ones. You may assume that $n \geq 3$.)

6. (15 points) In the flow network below, each edge from vertex i to vertex j is labeled with two numbers "$C_{ij} : F_{ij}$" where C_{ij} is the capacity of the edge and F_{ij} is an amount of flow on the edge, for a flow that is not a maximum flow. What is the widest augmenting path in this network, and what is its width? (You may find it helpful to construct the residual graph, but this is not a required part of your answer.)



7. (15 points) In the stable matching problem, define a matching to be "primary" if every match is the first choice of at least one participant. That is, for each matched pair of participants x and y, either x is y's first choice or y is x's first choice (or both). Suppose we are trying to match two people, Pat and Quinn, to job openings at two companies, Applied Infonautics and Bananabot. Both people prefer Applied Infonautics to Bananabot, and both companies prefer Pat to Quinn. Is there a matching that is both primary and stable? Why or why not?

8. (15 points) The proof of the four-color theorem can be turned into a polynomial time algorithm that takes as input a planar graph, and colors the vertices with four colors. (The lectures mentioned this algorithm but did not give any details of it.) However, some planar graphs may have colorings with fewer colors that this algorithm does not find. Consider an algorithm for finding approximate colorings of planar graphs that performs the following steps:

   - If the graph has no edges, use one color for all its vertices.
   - If the graph is bipartite, color it with two colors.
   - In all remaining cases, use the polynomial-time four-coloring algorithm.

   What is its approximation ratio?