ICS 163 – Spring 2002 – Final Exam

Name:

Student ID:

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Total:

1. Network reliability (30 points).

Suppose you are given as input a directed graph G and a pair of vertices s and t. Your task is to find a subgraph of G, with as few edges as possible, such that, even if a single edge is deleted, s and t can always be connected by a path using the remaining edges in the subgraph.

a. Describe how to formulate this problem as a min-cost flow. What are the edge capacities and costs in your flow formulation?

b. Why does the flow formulation give the optimal solution to this problem?

c. In the worst case, how many iterations would the shortest augmenting path method use to find the optimal flow? How much time would this method take?

2. Planar graphs (20 points).

If one tiles the plane with equilateral triangles, the result is an infinite planar graph with exactly six edges at every vertex. Can a finite planar graph have exactly six edges at every vertex? Why or why not?

3. Graph drawing (25 points).

Draw a tesselation representation of the following six-vertex *st*-planar graph. Use letters to label the objects in the drawing that correspond to vertices in the graph. Be careful of the directions of the arrows in the graph. Your drawing does not have to use the minimum possible number of rows or columns.

$$\begin{array}{ccccc}
 & & a & & \\
 & \nearrow & \uparrow & \nwarrow & \\
b & \rightarrow & c & \leftarrow & d \\
 & \nwarrow & \uparrow & & \uparrow \\
 & & e & \leftarrow & f
\end{array}$$

4. Exponential algorithms (?? points).

We discussed in class algorithms for finding a 3-coloring of a graph (if one exists), with the best running time being $O((3^{1/3})^n) \approx 1.44^n$. Describe an efficient exponential-time algorithm for finding a 4-coloring of a graph (if one exists). How much time does your algorithm use?

Special scoring rules: If you describe a correct algorithm with worst-case running time $O(c^n)$, your score will be $\lceil 100/c^2 \rceil$ points, so faster algorithms earn higher scores but non-optimal algorithms will still earn partial credit.

You may use this page for scratch paper.