Name: ID:

ICS 163 — Graph Algorithms — Winter 1994 — Final

1. Recall that an Euler tour of an undirected graph is a cycle in which each edge is used exactly once. A graph has an Euler tour iff every vertex has an even degree (number of adjacent edges).

   a. If a graph does not have an Euler tour, prove that it must have an even number of vertices with odd degrees.

   b. Describe an algorithm for finding the shortest cycle that uses each edge at least once, in a graph without an Euler tour. Hint: use a similar technique of matching odd vertices to the technique used in Christofides' heuristic for approximate travelling salesman tours.

Name: ID:

2. Suppose you are given a graph G for which the greedy heuristic colors G with 24 colors, and that you have also found a clique of 8 vertices in G. What if anything can you say about the approximation ratio of your coloring?

Name: ID:

3. For each of the following problems, state whether the problem is in P, whether it is NP-complete, or whether it is neither known to be in P nor known to be NP-complete.

   a. Finding a cycle that goes through each vertex exactly once.

   b. Finding a set of four independent vertices.

   c. Testing whether two given graphs are identical.

   d. Testing whether one given graph is a subgraph of another graph.

   e. Testing whether there is a path of K or fewer edges connecting a given pair of vertices.

Name: ID:

4. Describe an undirected graph for which the minimum spanning tree differs from the shortest path tree from some particular vertex. Show both trees for your graph.

Name: ID:

5. Recall that a graph G is k-vertex-connected if there is no set of $(k-1)$ vertices that disconnect G. A graph G is called *maximal planar* if G is planar, and if no edge can be added to G without making a crossing.

   a. Show that any maximal planar graph is 3-vertex-connected (hint: show that if any set S of fewer than 3 vertices disconnected $G - S$, it would be possible to add another edge connecting the pieces of $G - S$).

   b. Show that if some three vertices separate a planar graph G, it must be possible to add edges connecting those three vertices into a triangle.

   c. Use (b) to find an algorithm for testing whether a maximal planar graph is 4-vertex-connected. How fast does your algorithm run?

Name: ID:

6. Given a set of $m < n^2$ points in an $n \times n$ grid, the *escape problem* is to determine whether or not there are m vertex-disjoint paths from the starting points to the boundary. For instance the grid on the left below has an escape, but the grid on the right does not. (Similar problems come up in certain VLSI design situations.)

   Show how to translate the escape problem to a maximum flow problem. Hints: you will need to use directed edges to make sure that each vertex is only used by one path. It will work to set all edge capacities equal to one.