ICS 1F — Winter 1998 — Midterm

Name: SOLUTION KEY

Student ID:

Problem 1 (25 points possible):

Problem 2 (30 points possible):

Problem 3 (45 points possible):

Total (100 points possible):
1. State whether each of the following statements about sets A and B is always true, never true, or sometimes true and sometimes false depending on the particular sets represented by A and B.

(a) $A \in \wp(A)$  
\textit{always true}

(b) $A \subseteq \wp(A)$  
\textit{sometimes true (e.g. true for the empty set, false for the set \{1,2,3\})}

(c) $\overline{A \cap B} = \overline{A} \cup \overline{B}$  
\textit{always true (De Morgan's law)}

(d) $A \cup \overline{B} = \overline{A} \cup B$  
\textit{sometimes true (e.g. true if $A=B$, false if $A$ is the empty set and $B$ is nonempty)}

(e) $A \cup \overline{A} = \overline{B} \cap B$  
\textit{never true}

2. For the following problem, you may find it helpful to have a list of the first few powers of two. They are: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, and 1024.

(a) Convert the following three decimal numbers to binary: 15, 34, 770.

\begin{align*}
15: & \quad 1111 \\
34: & \quad 100010 \\
770: & \quad 1100000010
\end{align*}

(b) Convert the following three binary numbers to decimal: 1010, 11011, 10010100.

\begin{align*}
1010: & \quad 10 \\
11011: & \quad 27 \\
10010100: & \quad 148
\end{align*}
3. Consider the following deterministic finite automaton.

(a) Fill out the remaining entries in the following truth table.

<table>
<thead>
<tr>
<th>Input</th>
<th>Currently in state A:</th>
<th>Will be in state A after next transition:</th>
</tr>
</thead>
<tbody>
<tr>
<td>false</td>
<td>false</td>
<td>true</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td>true</td>
<td>false</td>
<td>true</td>
</tr>
<tr>
<td>true</td>
<td>true</td>
<td>false</td>
</tr>
</tbody>
</table>

(b) Draw a boolean circuit with two inputs and one output, that computes the function described by your truth table.

(c) Describe how to connect the outputs of your circuit back to its inputs to form a circuit having the same behavior as the automaton at the top of the page.