ICS 260 – Fall 2001 – First Midterm

Name:

Student ID:

1:

2:

3:

4:

5:

6:

Total:
1. Assembly line scheduling (20 points)

(a) In the following graph, boxes represent tasks that must be performed in the assembly of a computer, and arrows represent constraints that one task must be performed before another (for instance, the disk drive must be attached before the chassis can be covered). Write down a sequence in which all of these tasks can be performed, satisfying all constraints.

(b) Circle the phrase among the following that best describes an abstract algorithmic problem corresponding to this assembly line scheduling application.

- Breadth first search
- Depth first search
- Maximum flow
- Shortest paths
- Topological ordering
- Uniform packing

(c) If there are $t$ tasks and $c$ constraints, what is the best time bound for solving problems like the one in part (a)? Use $O$-notation.
2. Shortest paths. (20 points)
(a) Write down lengths for the edges of the following graph, so that Dijkstra’s algorithm would not find the correct shortest path from \( s \) to \( t \).

(b) Which of the shortest path algorithms described in class would be most appropriate for finding paths in the graph of part (a) with the weights you gave? Explain your answer.
3. Flow. (20 points)
(a) The numbers on the graph below represent edge capacities. Write down flow amounts for a maximum flow in this graph.

(b) Draw or describe the minimum cut for this graph.
4. Feasible circulation. (20 points)
(a) Translate the following feasible circulation problem into a maximum flow problem. Do not solve the flow problem. You may show your answer by adding vertices and edges to the given graph, and by writing capacities on each edge.

(b) Explain how you would determine whether there is a feasible circulation, if you were given the flow amounts of a maximum flow for your answer to part (a).
5. Preflow push algorithm. (20 points)
Suppose that we are trying to compute a maximum flow from s to t in the following graph (with indicated capacities) using the generic preflow-push algorithm.

(a) For the preflow amounts and distance labels shown in the tables below, fill in the column showing the excess at each vertex

<table>
<thead>
<tr>
<th>edge</th>
<th>flow amount</th>
<th>vertex</th>
<th>distance label</th>
<th>excess?</th>
</tr>
</thead>
<tbody>
<tr>
<td>s → A</td>
<td>6</td>
<td>s</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>s → B</td>
<td>2</td>
<td>A</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>A → B</td>
<td>2</td>
<td>B</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>A → C</td>
<td>2</td>
<td>C</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>B → t</td>
<td>3</td>
<td>t</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>C → t</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Describe one possible set of changes that could be made to the preflow amounts or distance labels of part (a) by a single step of the generic preflow-push algorithm.
6. Matching and independent sets. (20 points)

Would the maximum independent set algorithm described in the lecture on matching work for the following graph? Why or why not?
You may use this page for scratch paper.