ICS 261—Sample problems on segment and interval trees

David Eppstein, ICS, UC Irvine

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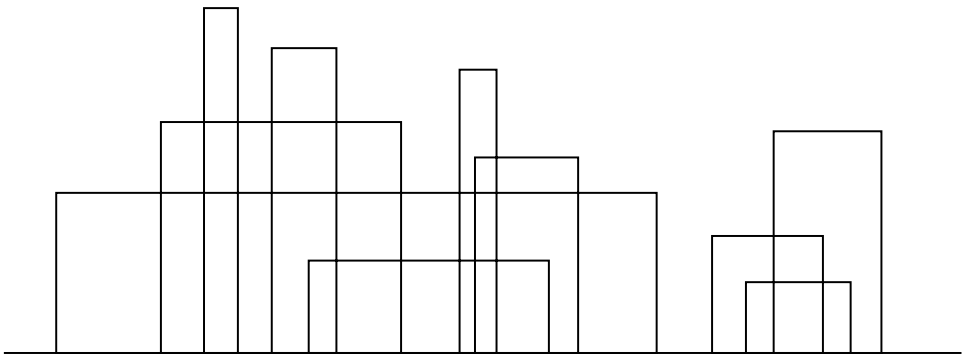
1. The "Manhattan Skyline" problem: suppose we are given a collection of rectangles, each of them having their bottom edge on the x axis, as in the figure below.



   Describe an $O(n \log n)$ time algorithm for computing the area of the union of the rectangles.

   Hint: sweep from left to right, using one of the data structures discussed in class to maintain the height of the skyline at each point of the axis.

2. Describe an $O(n \log n)$ time algorithm for computing the union of an arbitrary set of rectangles (not necessarily all having an edge on the axis).

   Hint: again sweep left to right, using a dynamic segment tree data structure to maintain the vertical extent of the set of intervals intersected by the sweep line.

3. Earlier in class we discussed a data structure based on persistent search trees for the "point location" problem: given a subdivision of the plane into polygons, quickly find the polygon containing a query point.

   (a) Describe an alternate algorithm for point location based on segment trees. What is the time and space complexity of your algorithm?

   Hint: build a segment tree on the set of intervals formed by the horizontal projections of the polygon edges. What data structure should you use to store the intervals in each canonical segment? Fractional cascading can be used to help improve the query time, but this requires a more detailed understanding of fractional cascading than I really went through in class, so the answer I'm looking for is just the basic uncascaded data structure.