Name:

Student ID:

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Total:
1. (18 points) Which element of the input is produced as the output of the streaming majority algorithm, when the input is the sequence A, A, B, A, B, C, D, A?

2. (24 points) Suppose you have a data set consisting of (key, priority) pairs. The five pairs in your data set are (A, 40), (B, 10), (C, 20), (D, 50), and (E, 30). Draw a tree for these pairs that is simultaneously a binary search tree (for the keys in alphabetical order) and a Fibonacci heap (a heap-ordered tree for the numerical priorities, ordered as a min-heap).

(Hints: first figure out which pair should be at the root and then work down. Be sure to draw your tree in a style that makes clear whether each node is a left child or a right child of its parent.)
3. (24 points) Suppose that, for some parameter \( n \), we perform \( \sqrt{n} \) insertions into a dynamic array data structure (and otherwise do not change the array). What is the worst-case total (actual) time for this sequence of insertion operations (in \( O \)-notation as a function of \( n \))

(a) ... assuming that the dynamic array starts out empty?

(b) ... assuming that the dynamic array starts out with at most \( n \) items already in it?

4. (24 points) For each of the following hash table data structures, state the maximum load factor \( f \) (number of keys divided by number of hash table cells) such that searches and insertions in the structure take constant expected time for load factors below \( f \), but not for load factors above \( f \). Your answer should be a number, not a formula. If the structure works in constant expected time for all constant load factors, write \( \infty \) as your answer.

(a) Hash chaining

(b) Linear probing

(c) Cuckoo hashing (with two locations per key and one key per cell)
5. (30 points) Suppose that sets $A$ and $B$ are both represented by separate Bloom filters, with tables of the same length as each other and with the same hash functions as each other. Describe how to compute from them a Bloom filter for their union, the set $A \cup B$. (You may describe this computation either in English text or pseudocode, as long as the description of the algorithm is clear.)
You may use this page (or the back of the other pages) as scratch paper.