Orientation-Constrained Rectangular Layouts

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Rectangular layout

partition of a rectangle into finitely many interior-disjoint rectangles, such that no four rectangles meet in one point.
RECTANGULAR DUALS

TRIANGULATED GRAPH $\mathcal{G}$

RECTANGULAR DUALS FOR $\mathcal{G}$
RECTANGULAR DUALS

TRIANGULATED GRAPH \( G \)

EXTENDED GRAPH \( E(G) \)

RECTANGULAR DUALS FOR \( E(G) \)

NOT A RECTANGULAR DUAL FOR \( E(G) \)
Rectangular cartograms visualize statistical data about sets of regions; regions are rectangles; area proportional to some geographic variable.
ORIENTATION CONSTRAINTS

PRESERVE RELATIVE POSITIONS OF THE REGIONS

CONSTRAINTS ON THE ORIENTATIONS OF THE ADJACENCIES
REGULAR EDGE LABELING (REL)
REGULAR EDGE LABELING (REL)
REL = RECTANGULAR DUAL

[Kant and He’97]

Every rectangular dual for $E(G)$ corresponds to a regular edge labeling of $E(G)$ and vice versa.
ORIENTATION-CONSTRAINED DUALS
SEPARATING 4-CYCLES

TRIVIAL SEPARATING 4-CYCLE

NON-TRIVIAL SEPARATING 4-CYCLE
DISTRIBUTIVE LATTICE OF RELS

[Fusy '05]
Birkhoff's Representation Theorem
Birkhoff's Representation Theorem
PARTIAL ORDER OF FLIPPABLE ITEMS

[ Eppstein, Mumford, Speckmann, Verbeek '09 ]
Birkhoff's Representation Theorem
ORIENTATION CONSTRAINTS

FORBID LABEL

UPWARD CLOSED

DOWNWARD CLOSED
Orientation Constraints

Forbidden label: \(b\rightarrow c\)

(a, 0)
(b, 0)
(c, 0)
(d, 0)
(e, 0)
(f, 0)
(g, 0)
(h, 0)
(i, 0)
(j, 0)
(k, 0)
(l, 0)
(m, 0)
(n, 0)
(o, 0)
(p, 0)
(q, 0)
(r, 0)
(s, 0)
(t, 0)
(u, 0)
(v, 0)
(w, 0)
(x, 0)
(y, 0)
(z, 0)
Orientation constraints

FORBID LABEL

\[ (bc, 1) \]

\[ (ab, 0) \]

\[ (ac, 0) \]

\[ (d, 0) \]

\[ (bc, 0) \]
Orientation constraints

FORBID LABEL
b
C

(b_c, 1)
(ab, 0)
(ac, 0)
(d, 0)
(bc, 0)
Orientation constraints

FORBID LABEL
\( \alpha \rightarrow \beta \)

\( (bc, 1) \)
\( (ab, 0) \)
\( (ac, 0) \)
\( (d, 0) \)
\( (bc, 0) \)
Orientation constraints

FORBID LABEL
$\text{$@\text{a}$} \rightarrow \text{$b$}$

Diagram showing various orientations and constraints with labels such as $bc$, $ab$, $ac$, and $d$.
Orientation constraints

FORBID LABEL
\(a \rightarrow b\)

\(d\)

\(ab\)

\(ac\)

\(bc\)

\(\infty\)

\((bc, 1)\)

\((ab, 0)\)

\((ac, 0)\)

\((bc, 0)\)

\(-\infty\)
Orientation Constraints

Forbidden Labels:

- \( a \rightarrow b \)
- \( b \rightarrow c \)

Network Diagram:

- Nodes: a, b, c, d
- Edges: ab, ac, bc, bd, cd

- \( +\infty \)
- \((bc,1)\)
- \((ab,0)\)
- \((d,0)\)
- \((ac,0)\)
- \((bc,0)\)
- \(-\infty\)
Results

- Find a layout for given constraints in polynomial time

- List all layouts satisfying the constraints in polynomial time per layout

- Find area-universal layouts satisfying the constraints