

Sample Questions: CS 177, 2008

Problem 1

(a) $f(X)$ is a probability density function where X is a continuous random variable. Define 2 properties that $f(X)$ must satisfy:

1:

2:

(b) Write down the equation for a linear congruential random number generator

(c) Consider a 4-state first-order Markov chain. Draw an example of a state diagram (transition diagram) for which steady-state probabilities can not be defined.

Problem 2:

Consider 2 random variables X (taking values 1, 2, or 3) and Y (taking values 0 or 1), with the following joint probability table:

		X		
		1	2	3
Y	0	0.1	0.4	0.2
	1	0.0	0.2	0.1

Answer the following questions. Where probability numbers are required you can express your answer in fractions:

(a) What is the probability of $P(X=1 | Y=1)$?

(b) What is the probability that X is greater than 1, given that $Y=0$

(c) What is the expected value $E[X]$

(d) What is the expected value $E[(X+1)(Y+2)]$

- (e) Are X and Y independent? Clearly show how you arrive at your answer.
- (f) Now say we have a 3rd variable Z. Say we know that Z is independent of X and that Z has probability $P(Z=0) = 0.4$ and $P(Z=1) = 0.6$. Write out the joint probability table for $P(X,Z)$.
- (g) What can we say about the dependence or independence of Y and Z given the information presented so far? explain your answer clearly.

Problem 3:

(a) State the Central Limit Theorem, clearly defining all quantities that you use in the definition of the theorem.

(b) X is a random variable that has a binomial distribution with parameters p and n . Define the binomial probability mass function. Explain clearly why it can be difficult to compute the binomial probabilities when n becomes large.

(b) Explain clearly how the Central Limit Theorem can be applied to the problem of computing the binomial probabilities in a binomial model.

- (e) Define the power method (in terms of initialization and each recursion) for computing the steady-state probabilities
- (f) If there are K non-zero entries in total in the $m \times m$ transition matrix P , what is the time-complexity per iteration of the power method?
- (g) Explain in 1 sentence why the answer to part (f) is directly relevant to the operation of the Google search engine

A arrives at time 0, departs at time 4
B arrives at time 2, departs at time 9
C arrives at time 5, departs at time 11
D arrives at time 7, departs at time 7
E arrives at time 10, departs at time 12

What was the service time for each customer? (if a customer was not served because of the finite queue length, indicate "not served").

4. What is average queue length over the first 10 time units (starting from time 0 when customer A arrives up until the end of time unit 10). Think carefully about how you answer this question.