CS177, Homework 8
Due Date: Wednesday, June 4th

Please upload your MATLAB functions and writeup to the appropriate EEE folder.

Reading

- Olofsson Section 5.4 on simulation methods
- Olofsson Section 7.4.3 and 7.4.4 on queueing systems

Problem 1: Simulating an infinite M/M/1 Queue

You are to implement an algorithm to simulate an M/M/1 queue with an infinite queue size. The input to your simulation function should be

- K, the number of customers to be simulated
- λ, the arrival rate
- μ, the service rate

It should return

- A list of K arrival times for the K customers (the first arrival can be at time 0 without loss of generality)
- A list of K service start times which specify when service began for each customer
- A list of K departure times (service end times)

Your function should have the following template

```
function [arrival_t, service_t, departure_t] = MM1_simulation(K,lambda,mu)
```

where the return values, e.g. `arrival_t`, are $Kx1$ arrays. Be sure to clearly document your code.

The first part of your simulation should be to generate the $K-1$ inter-arrival times and K service times. To accomplish this you will want to re-use the code you wrote in Homework 7 for sampling from an exponential density. To calculate the service start times for each customer, you will need to sequentially go through the arrival times while keeping track of the state of the system. When a new event happens (either a customer arrives or the server finishes serving a customer) you simulation code will need to determine the

new state of the system (e.g. when a service finishes and the queue isn't empty, remove the next customer from the queue and start serving them). Recall from class that the state of the system is described by how many people are in the system (both how many are in the queue and whether or not one is being served). The only time the state can change is either when the next person arrives or the server finishes serving someone, at all other times the state cannot change.

Problem 2: Simulating a finite length M/M/1 Queue

Write a function to simulate a finite queue of length $r-1$ (so there are no more than r total customers in the system at any given time, $r-1$ in the queue and 1 being served). With a finite queue, if the system is full when a new customer arrives, then that customer gets dropped and never gets served. You will want to start with your code from problem 1 but modify it to take a new parameter r which specifies the total number of customers allowed in the system. You also should return an additional boolean array of size $Kx1$ which indicates whether the kth customer got served or dropped.

Your function should have the following template

```
function [arrival_t,service_t,departure_t,served] = MM1_finite(K,lambda,mu,r)
```

Experiments

Please submit your 2 well documented MATLAB functions along with any other supporting code you wrote and a text document (txt/doc/pdf) with answers to the following questions:

1. For the infinite queue, generate histograms of both how long each customer spent waiting in the queue (W) and how much total time the customer spent in the system (T). You should generate the histograms with 50 bins using $K = 10000$ (or larger if you wish). Generate these two histograms for each of the following parameter settings (a) $\lambda = 1, \mu = 10$ (b) $\lambda = 5, \mu = 10$ (c) $\lambda = 9.9, \mu = 10$.

2. For each of the 3 cases in part 1, please compute the mean waiting times and total times from your simulation data. Briefly discuss why they are different across the 3 cases

3. Compare the mean values you computed in part 2 with the theoretical values given by the formulas for $E[T]$ and $E[W]$ that we derived in class. These should be close, if not then there is probably a bug in your simulator.

4. Use your finite queue simulator to generate histograms of W and T for $K = 10000$ samples. You should generate histograms with $\lambda = 0.9, \mu = 1$ and two values of the system size r, $r = 5$ and $r = 20$.

5. For these two different finite queues $r = 5, 20$, compute the percentage of customers who are turned away. Compare these to the theoretically predicted values we discussed in class and briefly discuss the differences between theory and simulation for the two different queue lengths.