

Oblivious RAM Simulation with Efficient Worst-Case Access Overhead

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ABSTRACT

Oblivious RAM simulation is a method for achieving confidentiality and privacy in cloud computing environments. It involves obscuring the access patterns to a remote storage so that others, including even the manager of that storage, cannot infer information about its contents. Existing solutions typically yield small amortized overheads for achieving this goal, but also yield huge variations in access times, depending on when they occur. In this paper, we show how to de-amortize oblivious RAM simulations, so that each access takes a worst-case bounded amount of time.

Categories and Subject Descriptors

K.6.5 [Management of Computing and Information Systems]: Security and Protection

General Terms

Algorithms, Security

1. INTRODUCTION

In the cloud storage model, a user we dub Alice remotely stores a large set of data with a remote server we dub Bob, offloading her need to maintain her data. To achieve data confidentiality from Bob, Alice should store her data in encrypted form, but encryption alone is insufficient, since information about her data may be leaked by the pattern in which she accesses it.

Oblivious RAM simulation tackles this privacy-protection problem by hiding data access patterns from Bob. Such solutions simulate a general random access machine (RAM) computation with respect to the external storage, but they perform additional obfuscating accesses that hide the locations of requests. Recently there has been a considerable amount of work on methods that optimize the *access overhead*, i.e., the number of additional accesses to the data repository per request to provide oblivious access (e.g., see [7, 6, 1, 12, 10, 2, 14, 5]). Nevertheless, existing oblivious RAM solutions still suffer from a worst-case access overhead that can be as bad as $\Omega(n)$ for n data items. This worst-case overhead makes oblivious RAM unsuitable in many practical scenarios, such as real-time systems and multi-user systems. Thus, in this paper

we are interested in efficient *de-amortized* solutions that have good worst-case bounds on the overhead for every request.

The Oblivious RAM Model. An *oblivious RAM (ORAM)* is an interface between a client Alice and a data repository Bob, whereby Alice outsources the storage of n data items to Bob. When Alice makes a *request* for item x , she issues a sequence of *accesses* to Bob's data repository to retrieve x in such a way that Bob is unable to determine which item is being accessed (any better than a random guess). Of course, there is a simple way for Alice to obfuscate her requests—she could simply read all items from the data repository with each request. Such a solution has access overhead $\Theta(n)$, which is quite inefficient.

Related Prior Work. Oblivious RAM models can be classified into *stateless* and *stateful* solutions. A stateless oblivious RAM is not allowed to keep a state between requests and hence can be used in a multi-user scenario. Stateful solutions assume Alice keeps information in a private storage (which she maintains), which helps her perform her accesses obliviously in the remote storage.

Stateless oblivious RAM simulation was first proposed by Goodrich and Ostrovsky in [5], who present a preliminary simple solution with $O(\sqrt{n} \log^2 n)$ amortized access overhead, referred as the *square-root solution*, and a more complex solution with $O(\log^3 n)$ amortized access overhead. Goodrich and Mitzenmacher [6] improve this result by giving a method with $O(\log^2 n)$ amortized access overhead with high probability. Recently Kushilevitz *et al.* [10] show that techniques from [6] can be extended to obtain $O(\log^2 n / \log \log n)$ amortized access overhead. All the above stateless methods utilize a private memory of size only $O(1)$ for Alice, an overly restrictive assumption in practice.

Other solutions [7, 13, 14] improve the overall access overhead by assuming that a client has a workspace of non-constant size. Williams and Sion [13] achieve $O(\log^2 n)$ expected amortized access overhead and $O(n \log n)$ space overhead with $O(\sqrt{n})$ private memory. Williams *et al.* [14] improve the method from [13] to achieve $O(\log n \log \log n)$ amortized access overhead. Goodrich *et al.* [7] give an oblivious RAM simulation method with $O(\log n)$ amortized access overhead given that a client has access to workspace of size $O(n^\nu)$, for a given constant $\nu > 0$.

Other recent papers provide stateful solutions, i.e., where a client maintains a state between requests to the data repository in a non-constant sized private cache. A RAM simulation by Goodrich and Mitzenmacher [6] achieves an overhead of $O(\log n)$ and uses a private cache of size $O(n^\nu)$, for any given fixed constant $\nu > 0$, which maintains a state. Boneh *et al.* [1] propose a scheme that achieves an amortized overhead of $O(1)$ but using a cache of size $O(\sqrt{n} \log n)$, which also maintains state.

Damgård *et al.* [2] and Goodrich *et al.* [7] present stateless oblivious RAM simulations without cryptographic assumptions

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CCSW'11, October 21, 2011, Chicago, Illinois, USA.

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Table 1: Comparison of Oblivious RAM simulation Methods.

	Client Memory	Server Storage Overhead	Amortized Access Overhead	Worst-Case Access Overhead
Goldreich-Ostrovsky [5] \sqrt{n}	$O(1)$	$O(n)$	$O(\sqrt{n} \log^2 n)$	$O(n \log^2 n)$
Goldreich-Ostrovsky [5] $\log n$	$O(1)$	$O(n \log n)$	$O(\log^3 n)$	$O(n \log^2 n)$
Williams <i>et al.</i> [14]	$O(\sqrt{n})$	$O(n \log n)$	$O(\log^2 n)$	$O(n \log n)$
Goodrich-Mitzenmacher [6]	$O(1)$	$O(n)$	$O(\log^2 n)$	$O(n)$
Kushilevitz <i>et al.</i> [10]	$O(1)$	$O(n)$	$O(\log^2 n / \log \log n)$	$O(n)$
Stefanov <i>et al.</i> [12]	$O(\sqrt{n})$	$O(n)$	$O(\log^2 n)$	$O(\sqrt{n})$
Goodrich-Mitzenmacher [6]	$O(n^\nu)$	$O(n)$	$O(\log n)$	$O(n)$
Goodrich <i>et al.</i> [7]	$O(n^\nu)$	$O(n)$	$O(\log n)$	$O(n)$
Boneh <i>et al.</i> [1]	$O(\sqrt{n} \log n)$	$O(n)$	$O(1)$	$O(n \log n)$
De-amortized $\log n$	$O(n^\tau)$	$O(n)$	$O(\log n)$	$O(\log n)$
De-amortized \sqrt{n}	$O(1)$	$O(n)$	$O(\sqrt{n} \log^2 n)$	$O(\sqrt{n} \log^2 n)$

about the existence of random hash functions. Damgård *et al.* [2] show that amortized access overhead of $O(\log^3 n)$ is possible for oblivious RAM simulation without using random functions. Goodrich *et al.* [7] present a method with $O(\log^2 n)$ amortized access overhead that also does not use random functions.

To sum up, all the methods described above have amortized access overheads. (See Table 1 for the comparison of ORAM simulations.) Indeed, even the most efficient previous solutions can incur an $O(n)$ overhead in the worst case for any given request. This is slightly improved in recent work by Stefanov *et al.* [12], who give an oblivious RAM simulation that achieves an $O(\sqrt{n})$ access overhead in the worst case, while having $O(\log^2 n)$ amortized overhead complexity; hence, their solution is also amortized, but not as inefficient as previous schemes on a per-access basis.

Kosaraju and Pop [9] give an overview of general de-amortization techniques. E.g., one of the techniques, de-amortization via data duplication [3], maintains two copies of the data set: one for performing the redistribution of the data and one for accesses. We cannot apply these general techniques to our problem, however, since we also need to ensure the obliviousness of the de-amortized algorithm.

Our Results. We present two oblivious RAM simulations that achieve a sublinear access overhead on every request made by the client to the data repository. The first is a de-amortized version of the square root solution originally presented in [5]. This method has $O(\sqrt{n} \log^2 n)$ access overhead in the worst case while using $O(n)$ space for the data repository and assumes only $O(1)$ workspace on the client side. We then de-amortize an efficient oblivious RAM simulation by Goodrich *et al.* [7], that we refer to as the “log n hierarchical” solution. In this solution, we achieve worst-case access overhead of $O(\log n)$ and space overhead of $O(n)$, assuming that a client has access to a workspace of size $O(n^\tau)$, for any given fixed constant $\tau > 0$.

2. PRELIMINARIES

We assume that the client outsources n data items to a remote data repository that supports the following *access operations*:

- *read*(i): return the content of location i ;
- *write*(i, x): write data item x to location i ;
- *copy*($i, j, count$): copy a memory block of size *count* from location i to location j .

The latter block-copy operation is not actually required by our methods but using it makes the algorithms more intuitive.

We also assume that provider of the storage service, Bob, is an *honest-but-curious* adversary [4], in that he correctly performs all operations and does not tamper with the data.

Alice stores a data item in the repository as the encryption of pair (x, v) , where x is the virtual address of the item in the RAM and

v is its value. Typically, oblivious RAM solutions use *probabilistic encryption* to make sure that Bob cannot distinguish between reads and writes or track repeated accesses to the same data item. Namely, Alice encrypts each data item that she writes to the data repository using a probabilistic encryption scheme based on her private key. Also, Alice reencrypts and rewrites each data item she accesses so that its encryption changes even if the data item is not modified. This technique ensures that Bob is computationally unable to determine the plaintext of any memory cell from that cell’s contents alone. Also, it is unfeasible for Bob to determine whether two memory cells store encryptions of the same data item.

Square-Root Solution. We first give an overview of the square-root oblivious RAM simulation method [5], which has $O(\sqrt{n} \log^2 n)$ amortized access overhead. We give enough details about the method for a reader to understand our de-amortized version provided in Section 3.

The square-root solution uses storage space of size $n + 2\sqrt{n}$ at the data repository. (See Figure 1(a).) This space is split into a buffer, B , and a table, T . The buffer B has size \sqrt{n} and is used to cache the last \sqrt{n} requests. Table T contains a pseudo-random permutation of the n data items and \sqrt{n} dummy items.

Each data item is associated with a key (virtual RAM address) x , $x = 1, \dots, n$ and each dummy item is given a key $n + d$ where $d = 1, \dots, \sqrt{n}$. All items in T are ordered according to a pseudo-random permutation function π such that $\pi(x)$ gives the location of the item with key x in T . (The full solution, which uses a binary search, is omitted in our description since the overall complexity of the request is $O(\sqrt{n})$.) The square-root ORAM simulation method is outlined in Algorithm 1. Note that table T has to be rebuilt after every \sqrt{n} requests. The rebuild phase consists of obviously replacing the items in T for which there is a new instance in B , associating the keys of real and dummy items with tags from a new permutation π' , and sorting items in T according to π' . The rebuild phase takes $O(n \log^2 n)$ accesses. Since the rebuild happens only once every \sqrt{n} requests, the amortized access overhead per request is $O(\sqrt{n} \log^2 n)$: $O(\sqrt{n})$ accesses for the request phase and $O(\sqrt{n} \log^2 n)$ accesses for the rebuild phase.

Requests are handled by scanning buffer B and accessing T . Due to the scheduled rebuilds, data items are associated with new tags every \sqrt{n} requests. Between the rebuilds, unique locations are accessed in T : either an item x is not present in the buffer and hence a unique location, $\pi(x)$, is accessed, or a unique dummy item, $\pi(n + request_count)$, is accessed.

log n Hierarchical Solution. We now describe the efficient oblivious RAM simulation by Goodrich *et al.* [7]. This method has $O(\log n)$ amortized access overhead, $O(n)$ space overhead at the repository, and client memory of size $O(n^\nu)$, where ν is an arbitrary positive constant. This method is stateless and thus

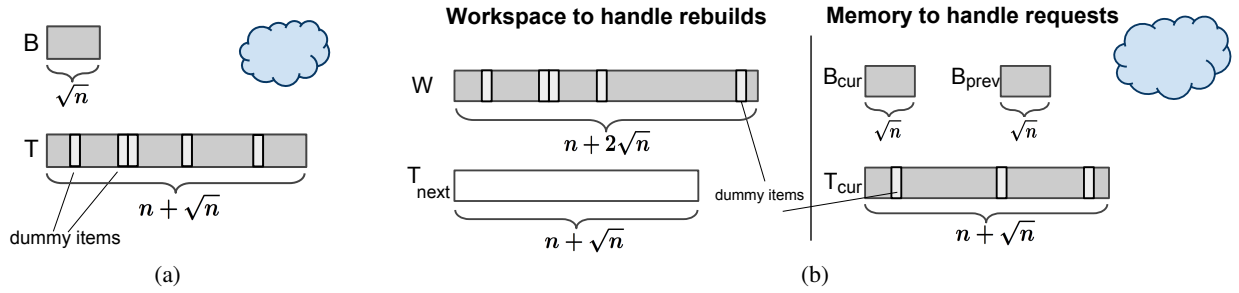


Figure 1: Memory layout of the data repository during oblivious RAM simulation: (a) original version of the square-root solution [5]; (b) our de-amortized version (Section 3).

suitable for a multi-user scenario since no client keeps a state between requests. Since the method of [7] uses *cuckoo hash tables*, we give a short description of them. (See [11] for more details.)

A *cuckoo hash table* T for n items consists of two hash tables H_1 and H_2 with two hash functions, h_1 and h_2 . Each hash table contains $m = (1 + \epsilon)n$ memory cells, with ϵ a (small) positive constant. An item with key x is found either in location $h_1(x)$ in H_1 or in $h_2(x)$ in H_2 . We say that a failure occurs when an item cannot be placed; for example, the most common failure is that there are only two possible locations for three keys x, y, z , i.e. $h_1(x) = h_1(y) = h_1(z)$ and $h_2(x) = h_2(y) = h_2(z)$. In this case a stash, S , of constant size is used to keep items that did not find a place in tables H_1 and H_2 . The retrieval of an item now includes a scan of S , which takes $O(1)$ time. Kirsch *et al.* [8] show that on any insertion a cuckoo hash table with a stash of size s overflows with probability $O(1/n^{s+2})$. Goodrich and Mitzenmacher [6] show that one can construct a cuckoo hash table of size n with a stash of size s obliviously using $O(n + s)$ accesses to the data repository and assuming access to private workspace of size $O(n^\nu)$.

In the method by Goodrich *et al.* [7], the memory at the data repository consists of a cache C for q data items, cuckoo hash tables T_1, \dots, T_L and a stash S . (See Figure 2(a).) The size of the cache, q , is $O(\log n)$ and stash S has also size $O(\log n)$. Each table T_i has size $2^i q$ and L is the first integer i such that $|T_i| \geq n$, hence L

Algorithm 1 Oblivious RAM simulation using the square-root approach [5].

```

Generate pseudo-random permutation function  $\pi$ 
Initialize table  $T$  by storing the  $n$  data items and  $\sqrt{n}$  dummy items according to permutation  $\pi$ 
request_count  $\leftarrow$  1
while true do {process a request}
  found  $\leftarrow$  false
  Scan all the locations in buffer  $B$ . During the scan, if data item  $x$  is found, set found  $\leftarrow$  true.
  if found then
    Access location  $\pi(n + \text{request\_count})$  in  $T$  {dummy item}
  else
    Access location  $\pi(x)$  in  $T$  {data item  $x$ }
  end if
  Rewrite  $B$ , adding or replacing data item  $x$ 
  request_count  $\leftarrow$  request_count + 1
  if request_count  $>$   $\sqrt{n}$  then
    Generate pseudo-random permutation function  $\pi'$ 
    Construct a new table  $T'$  with  $\pi'$  using items in  $T$  and  $B$ .
    clear  $B$  and set  $T \leftarrow T'$ ,  $\pi \leftarrow \pi'$ , and request_count  $\leftarrow$  1
  end if
end while

```

Algorithm 2 The request phase during oblivious RAM simulation with the $\log n$ hierarchical approach [7].

```

found  $\leftarrow$  false
scan cache  $C$  and stash  $S$ . if  $x$  is found in one of them set found  $\leftarrow$  true
for each level  $i$ ,  $1 \leq i \leq L$  do
  if found is true access random locations in  $T_i$ .
  else access locations  $h_1^i(x)$  and  $h_2^i(x)$  in  $T_i$ .
  if  $x$  is found set found  $\leftarrow$  true
end for
Remove  $x$  from  $S$  if  $x$  was found in  $S$ . Rewrite  $S$ .
Rewrite  $C$ , adding or replacing data item  $x$ .

```

is $O(\log n)$. Each cuckoo hash table T_i is initialized with two hash functions, h_1^i and h_2^i . The stash S is shared between all L tables and is large enough to avoid overflows in tables T_1, T_2, \dots, T_L with high probability [7].

On a request for item x , the client executes Algorithm 2. After q requests cache C becomes full and we obliviously move elements from C to T_1 . The move consists of creating a new cuckoo hash table T_1 from the elements in C . The next time C becomes full, instead of moving C to T_1 , we move the items from C and T_1 to T_2 . Similarly, when we are about to move items to T_i for the second time, instead we move all items from C, T_1, \dots, T_{i-1} to the first empty table among T_i through T_L . Oblivious construction of cuckoo hash table T_i takes $O(|T_i|)$ accesses to the data repository. Since table T_i is rebuilt every $O(|T_i|)$ requests and eventually every request causes $O(\log n)$ tables to be rebuilt, the amortized overhead is $O(\log n)$ accesses per request. The scans and writes to cache C and stash S are oblivious since they are done sequentially.

Table T_i is accessed either according to a pseudo-random hash function or in random locations because item x is found in an earlier level of the construction. Cache C and tables $T_j, j < i$, are empty when T_i is rebuilt and hence are used to cache items found in T_i . Since T_i is emptied and initialized with two new pseudo-random hash functions as soon as levels above it become full, it is never accessed twice for the same item. Hence, from the point of view of the adversary, Bob, accesses to T_i look random.

3. DE-AMORTIZED ORAM SIMULATION

The Square Root Solution. We present first the simple square-root solution, to demonstrate some of the ideas behind the more efficient technique developed in Section 3. Note that only recently an oblivious RAM solution with sublinear worst-case access overhead has been proposed [12].

Intuition: The most expensive step of the square-root approach is building a new table, where items and dummy values are ordered using a new pseudo-random permutation. This step is executed every \sqrt{n} requests and takes $O(n \log^2 n)$ accesses. Our idea is

to split the accesses for the rebuild into \sqrt{n} batches, each with $O(\sqrt{n} \log^2 n)$ accesses, and to execute each batch after processing a request so that the new table is ready to be used after processing \sqrt{n} requests. We will show how this idea can be implemented while preserving obliviousness and keeping the same asymptotic access overhead and storage overhead as the original method.

Memory Layout: We organize the memory on the data repository into five areas. We make use of two buffers, B_{cur} and B_{prev} , each of size \sqrt{n} . We also have two tables, T_{cur} and T_{next} , each of size $n + \sqrt{n}$. These tables are built using different pseudo-random permutations on the n data items outsourced by the client and \sqrt{n} dummy values. Finally, we employ a workspace W of size $n + 2\sqrt{n}$ for constructing incrementally the new table, T_{next} , while the current table, T_{cur} , and the two buffers, B_{cur} and B_{prev} , are being used to process requests. (See Figure 1(b) for illustration.)

Initialization: We split a sequence of requests into *epochs*, each consisting of \sqrt{n} requests. Initially, buffers B_{cur} and B_{prev} are empty and each of the tables T_{cur} and T_{next} contains the n items and \sqrt{n} dummy items permuted according to a pseudo-random permutation, where T_{cur} uses permutation π_0 and T_{next} uses permutation π_1 .

Processing an Epoch: Buffer B_{cur} caches the \sqrt{n} items being requested in the current epoch while buffer B_{prev} caches the \sqrt{n} items that were requested in the previous epoch. Thus, B_{prev} is empty during the first epoch. Also, table T_{cur} is used for processing requests and workspace W is used to build incrementally a new table, based on a new pseudo-random permutation. At the end of an epoch, the new table is copied from W to T_{next} . Next, table T_{cur} and buffer B_{cur} are copied to W . Finally, table T_{next} is copied to T_{cur} to accommodate the requests from the next epoch. Also, we overwrite B_{prev} with items from B_{cur} and we empty B_{cur} .

Incremental Table Construction: The construction of the new table, T_{next} in workspace W takes as input T_{cur} and B_{cur} from the previous epoch. We say that the instance of a data item in T_{cur} is stale if there is an instance of the same data item in B_{cur} . Using an algorithm from [5], we obviously filter out the stale instances of the data items and we construct a table for the set consisting of the n data items and \sqrt{n} dummy items, storing them according to newly generated pseudo-random permutation. Since this algorithm performs $O(n \log^2 n)$ accesses to the data repository, we de-amortize it by splitting its sequence of accesses to workspace W into \sqrt{n} batches of $c\sqrt{n} \log^2 n$ accesses each, for some constant $c > 0$. The construction of table T_{next} starts at the beginning of the epoch and a batch of accesses is executed after processing each request. Hence, the new table T_{next} is ready by the end of the epoch.

Our oblivious RAM simulation algorithm based on the square-root approach is outlined in Algorithm 3 and its properties are summarized in Theorem 1.

THEOREM 1. *Our oblivious RAM simulation method based on the square-root approach has $O(\sqrt{n} \log^2 n)$ worst-case access overhead per request, $O(\sqrt{n})$ space overhead at the data repository, and $O(1)$ client memory, where n is the number of data items.*

PROOF. (SKETCH) The worst-case access overhead of each request is $O(\sqrt{n} \log^2 n)$ since we scan two buffers of size \sqrt{n} , access one table entry, and execute $O(\sqrt{n} \log^2 n)$ accesses to perform one batch of the table rebuild. Also, $O(n)$ additional space is used at the server. We now consider the obliviousness of our method. During each epoch, unique items are accessed in table T_{cur} . Namely, if the requested data item is not found in the buffer, we access it in T_{cur} , else we access a new dummy item in T_{cur} . Moreover, in the beginning of each epoch T_{cur} is initialized with a new permutation over n items and \sqrt{n} dummy values. The

Algorithm 3 Oblivious RAM simulation with our de-amortized version of the square-root approach.

```

Generate pseudo-random permutation function  $\pi_{\text{cur}}$ 
Initialize table  $T_{\text{cur}}$  by storing the  $n$  data items and  $\sqrt{n}$  dummy
items according to permutation  $\pi_{\text{cur}}$ 
Initialize  $W$  with  $n$  data items and  $\sqrt{n}$  dummy items
while true do {process the requests in an epoch}
    Generate pseudo-random permutation function  $\pi_{\text{next}}$ 
     $request\_count \leftarrow 1$ 
    while true do {process request for data item  $x$ }
         $found \leftarrow false$ 
        Scan all the locations in buffers  $B_{\text{cur}}$  and  $B_{\text{prev}}$ . During
        the scan, if data item  $x$  is found, set  $found \leftarrow true$ .
        if found then
            Access location  $\pi_{\text{cur}}(n + request\_count)$  in  $T_{\text{cur}}$  {con-
            taining a dummy item}
        else
            Access location  $\pi_{\text{cur}}(x)$  in  $T_{\text{cur}}$  {containing data item  $x$ }
        end if
        Rewrite  $B_{\text{cur}}$ , adding or replacing data item  $x$ 
        Execute the next batch of  $c\sqrt{n} \log^2 n$  accesses to
        workspace  $W$  to construct table  $T_{\text{next}}$  using permutation
         $\pi_{\text{next}}$ 
         $request\_count \leftarrow request\_count + 1$ 
        if request\_count >  $\sqrt{n}$  then
            break {end of the epoch}
        end if
    end while
Copy the new table from  $W$  to  $T_{\text{next}}$ 
Copy  $B_{\text{cur}}$  to  $B_{\text{prev}}$ 
Copy  $T_{\text{cur}}$  and  $B_{\text{cur}}$  to  $W$ 
Empty  $B_{\text{cur}}$ 
Copy  $T_{\text{next}}$  to  $T_{\text{cur}}$ 
 $\pi_{\text{cur}} \leftarrow \pi_{\text{next}}$ 
end while

```

method is correct since the user is always returned the most up-to-date instance of the requested item: if the requested data item was last requested in the current epoch then it is found in B_{cur} , else if it was last requested in the previous epoch, it is found in B_{prev} , else T_{cur} has the latest instance. \square

Read/Write Data Repository: In Algorithm 3 we assumed that the data repository allows us to manage outsourced memory using a *copy* operator. However, we can achieve the same worst case overhead of $O(\sqrt{n} \log^2 n)$ without this operator. If the data repository supports only read and write operations one can alternate blocks of memory used for rebuilding and for handling requests between the epochs, e.g. during even numbered epochs B_{cur} is used to cache current requests while during odd epochs it serves as a buffer of requests from the previous epoch.

The log n Hierarchical Solution. The intuition behind the de-amortized version of this method is similar to that for the square-root solution: we incrementally rebuild tables while handling requests using previous versions of tables and buffers. Requests are handled using two sets of buffers: one for items requested in the current epoch and the other for items requested in the previous epoch. However, recall that the construction of [7] has $O(\log n)$ buffers implemented as cuckoo hash tables. This complicates our task since now we need to have a copy of each cuckoo hash table. Also due to the dynamic arrangement of the buffers, one buffer

spills into the next one and so on. We eventually need to construct $O(\log n)$ cuckoo hash tables during each epoch.

Memory Layout: Our memory layout at the data repository is schematically illustrated in Figure 2(b). Extending the oblivious RAM data structure of [7] (see Section 2), we employ two caches, C_{cur} and C_{prev} , of size $q = O(\log n)$, one stash, S , of size $O(\log n)$, and $2L - 1$ cuckoo hash tables $T_1, T'_1, T_2, T'_2, \dots, T_{L-1}, T'_{L-1}, T_L$ where each T_i and T'_i has size $2^i q$ and L is the smallest i such that $2^i q \geq n$. We also keep a workspace W for rebuilding cuckoo hash tables. The workspace stores the last $2^L q$ requested items in a list, D . In addition, it contains L work areas for rebuilding cuckoo tables. The i -th work area consists of storage space T_i^W of size $O(2^i q)$ for a cuckoo hash table at level i , and of overflow space S_i^W of size $O(\log n)$ for the corresponding stash.

Initialization: We build T_L as a cuckoo hash table for the n data items and put into stash S any items that did not fit. Both caches C_{cur} and C_{prev} and other tables T_i and T'_i (for $i < L$) are empty.

Processing an Epoch: In this section an epoch is defined as a sequence of q requests. During an epoch, cache C_{cur} stores data items last requested in the current epoch and cache C_{prev} stores data items last requested in the previous epoch. Thus, these caches play roles similar to those of buffers B_{cur} and B_{prev} in Section 3. Each request is processed by scanning caches C_{cur} and C_{prev} , scanning stash S , and accessing locations in tables $T_1, T'_1, T_2, T'_2, \dots, T_{L-1}, T'_{L-1}, T_L$. In addition, a batch of accesses is made to workspace W toward rebuilding its cuckoo tables. The incremental rebuilding process guarantees the completion of a cuckoo table in T_i^W and its stash in S_i^W after 2^{i-1} epochs.

Incremental Construction of L Cuckoo Hash Tables: Recall that a cuckoo hash table T_i of size $2^i q$ can be constructed obliviously using $2^i q$ accesses to the data repository and $O(n^\nu)$ private memory [6]. To help us explain the concurrent oblivious rebuild of L cuckoo hash tables, we introduce a data structure I that stores the set of indices of the cuckoo tables that need to be rebuilt in workspace W . Note that $Queue$ depends only on the number of requests made so far hence it can be computed in constant time. I starts empty. After every 2^{i-1} epochs index i is added to I . When an index i is added to I a sequence of $2^i b q$ accesses is required for a rebuild of T_i^W , for some constant $b > 0$. After each request, the client executes $2b$ accesses for each index in I so that the construction of table T_i^W is completed in 2^{i-1} epochs. Observe that after the first 2^{i-1} epochs index i is always present in I . Moreover, after 2^{L-1} epochs indices $1, 2, \dots, L$ are present in I and I does not change from then on. This also means that eventually all L tables are being rebuilt during an epoch. To accommodate L concurrent rebuilds, we increase the requirement on the size of client's private memory from $O(n^\nu)$ in [7] to $O(n^\tau)$, for some fixed constants $\tau > \nu$ and $\nu > 0$.

Transitioning to the Next Epoch: We append items in C_{cur} to D , the list in workspace W that keeps track of the $2^L q$ previously requested items. We then copy C_{cur} to C_{prev} and empty it. Hence C_{cur} can be used to cache requests during the next epoch. We then check which tables are finished, i.e. T_i^W is finished if the current number of epochs is a multiple of 2^{i-1} since T_i^W takes 2^{i-1} epochs for a rebuild. Each such table T_i^W is copied to either T_i or T'_i . If T_i and T'_i are both empty or both full T_i^W is copied to T_i , stash S_i^W is merged with S and T_i is cleared. If only T_i is full T_i^W is copied to T'_i , stash S_i^W is merged with S . If T_L is finished we clear first $2^L q$ items from D since all these items are now in T_L and no table from earlier levels requires them for a rebuild.

Stash Size: Goodrich *et al.* [7] show that a single stash of size $O(\log n)$ is enough to avoid overflows in cuckoo hash tables T_1, \dots, T_L where T_i contains $2^i q$ items. In our construction, we

use a single stash S for two collections of cuckoo hash tables. A single stash ensures that if item x happened to not fit into two tables T_i and T'_j then only the most recent copy is present in S . One can view stash S as a joint stash between tables T_1, \dots, T_L and T'_1, \dots, T'_{L-1} . Suppose a stash of size $s \log n$ is used in the construction of [7], where $s > 1$ is a constant. Then we set our stash S to be of size $2s \log n$, where the first $s \log n$ locations are used for tables T_1, \dots, T_L and the remaining $s \log n$ locations for T'_1, \dots, T'_{L-1} , with the additional constraint that only unique items can be present in S . The latter constraint is enforced when we merge stash S_i^W of a new table T_i^W with S .

Our oblivious RAM simulation algorithm based on the $\log n$ hierarchical approach is outlined in Algorithm 4 and its properties are summarized in Theorem 2.

THEOREM 2. *Our oblivious RAM simulation method based on the $\log n$ hierarchical approach has $O(\log n)$ worst-case access*

Algorithm 4 Oblivious RAM simulation with our de-amortized version of the $\log n$ hierarchical approach.

```

Initialize  $T_L$  and  $S$  with a cuckoo hash table with a stash using
 $n$  data items.
 $I \leftarrow \{\}$ ,  $request\_count \leftarrow 0$ 
while true do
  while true do {on request  $x$ }
     $found \leftarrow false$ 
    Scan all the locations in caches  $C_{\text{cur}}$  and  $C_{\text{prev}}$  and stash
     $S$ . During the scan, if data item  $x$  is found,  $found \leftarrow true$ .
    for each level  $i$ ,  $1 \leq i \leq L$  do
      if  $i \neq L$  and  $T'_i$  is not empty
        if  $found$  is true access random locations in  $T'_i$ .
        else access locations  $h_1^{i'}(x)$  and  $h_2^{i'}(x)$  in  $T'_i$ .
        if  $x$  is found,  $found \leftarrow true$ .
      if  $T_i$  is not empty
        if  $found$  is true access random locations in  $T_i$ .
        else access locations  $h_1^i(x)$  and  $h_2^i(x)$  in  $T_i$ .
        if  $x$  is found,  $found \leftarrow true$ .
    end for
    Rewrite  $C_{\text{cur}}$ , adding or replacing data item  $x$ .
    if  $x$  is found in stash  $S$  remove  $x$  from  $S$ . Rewrite  $S$ .
    for  $i \in I$ 
      Make next  $2b$  accesses towards a rebuild of table  $T_i^W$ 
       $request\_count \leftarrow request\_count + 1$ 
    if  $request\_count \bmod q = 0$  then
      {end of the epoch}
      Copy  $C_{\text{cur}}$  to  $C_{\text{prev}}$  and append it to  $D$ .
      Empty  $C_{\text{cur}}$ .
      for  $i \in sorted\_decr\_order(I)$ 
        if  $request\_count \bmod 2^{i-1} q = 0$ 
          if  $i = L$  copy  $T_i^W$  to  $T_L$ .
          else if  $T_i$  and  $T'_i$  are both full or both empty
            Empty  $T'_i$  and copy  $T_i^W$  to  $T_i$ .
          else copy  $T_i^W$  to  $T'_i$ .
          Merge  $S_i^W$  and  $S$ .
        for each level  $i$ ,  $1 \leq i \leq L$ 
          if  $request\_count \bmod 2^{i-1} q = 0$ 
            Copy last  $2^{i-1} q$  items from  $D$  to  $T_i^W$ 
            if  $i \notin I$  set  $I \leftarrow I \cup \{i\}$ .
            if  $i = L$  empty  $D$ .
        end if
      end while
    end while

```

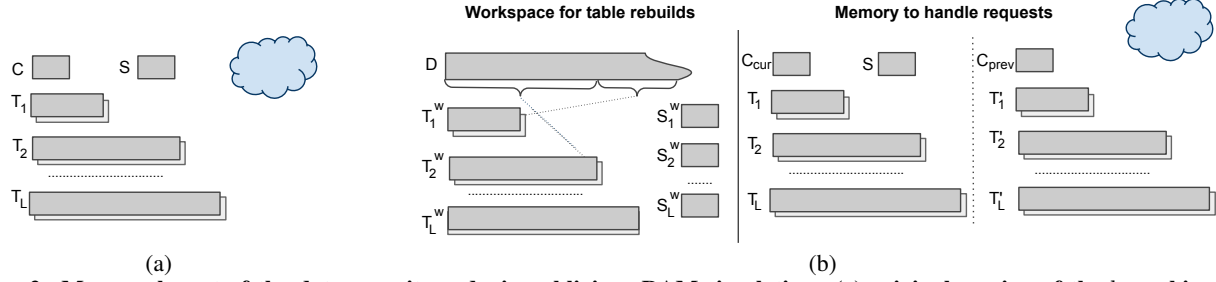


Figure 2: Memory layout of the data repository during oblivious RAM simulation: (a) original version of the $\log n$ hierarchical solution [7]; (b) our de-amortized version (Section 3).

overhead per request, $O(n)$ space overhead at the data repository, and $O(n^\tau)$ client memory, where n is the number of data items and τ is any fixed positive constant.

PROOF. (SKETCH) We first show that handling of each request using the above protocol takes $O(\log n)$ accesses. Retrieving a data item takes $O(\log n)$ accesses since three blocks of size $O(\log n)$ are scanned and two accesses are made to $2L$ tables, where L is $O(\log n)$. The batch of accesses for table rebuilding made after each request consists of $2b$ accesses for every table in I , where I has at most L indices and b is a constant. For every rebuild, we use the method of [6] which requires $O(n^\nu)$, $\nu > 0$, of client private memory. Since our method concurrently makes $O(\log n)$ rebuilds $O(n^\tau)$ of private memory is required for $\tau > \nu$.

We now consider the obliviousness of the method. Table rebuilds remain oblivious since they follow a predetermined schedule that depends on n and $request_count$ and are performed in the same way as in the original ORAM construction in [7].

It remains to show that each request remains oblivious. Accesses to the caches, C_{cur} and C_{prev} , and the stash, S , are oblivious since their memory locations are scanned (read and rewritten entirely) for each request. Since accesses to table T_i depend on whether an item is found in T_i' we first show the obliviousness of access sequence to T_i' . Observe that when T_i' is substituted with a new cuckoo hash table T_i^W cache C_{cur} and all tables $T_{j < i}'$ are empty. Since each table on level i can store up to $2^{i-1}q$ items before it is emptied there is space to remember the following number of requests:

$$q + \sum_{j=1}^{i-1} 2^{j-1}q = 2^{i-1}q.$$

If an item is not found in previous levels, it is accessed according to pseudo-random hash functions $h_1^{i'}$ and $h_2^{i'}$. Otherwise, T_i' is accessed in random locations. T_i' is cleared as soon as next table for this level, i.e. next T_i^W , is ready. This happens $2^{i-1}q$ requests after T_i' was last substituted with a new table. Hence, T_i' is never accessed more than once for the same item.

An access to table T_i follows an access to T_i' and is random if an item is found in earlier tables or in T_i' . Note that similarly to T_i' cache C_{cur} and all tables $T_{j < i}'$ are empty when T_i is ready. Moreover T_i' is empty as well. Hence, there is space to remember the following number of requests:

$$q + \sum_{j=1}^{i-1} 2^{j-1}q + 2^{i-1}q = 2^i q.$$

However, T_i is replaced with a new table every $2^i q$ requests. Hence, no location is accessed more than once in table T_i as well.

To prove the correctness of the method, we observe that the most current copies of the data items are present in the caches or smaller tables. Moreover, table T_i' contains more recent requests than T_i and stash S contains any items that did not fit in their corresponding tables. When newly constructed tables are moved from W to the memory for handling requests, we merge the stash of larger tables with S first. In this case, if the same item did not fit into more than one table, only the most recent copy is in S . Since we first scan the caches, the stash and start accessing tables from smaller levels,

with T_i' before T_i , our method returns to the user the latest instance of the requested item. \square

Read/Write Data Repository: Similarly to the de-amortized version of the square root solution from Section 3, we can relax the assumption on the interface that data repository provides us. If read and write are the only supported operations, we can alternate the blocks of memory used for rebuilds and for handling the requests depending on the epoch count.

4. ACKNOWLEDGEMENTS

Work supported in part by NSF grants 0721491, 0847968, 0915922, 0953071, 0964473, 1011840, and 1012060.

5. REFERENCES

- [1] D. Boneh, D. Mazieres, and R. A. Popa. Remote oblivious storage: Making oblivious RAM practical. Technical report 2011-018, CSAIL, MIT, 2011.
- [2] I. Damgård, S. Meldgaard, and J. B. Nielsen. Perfectly secure oblivious RAM without random oracles. In *TCC*, 144–163, 2011.
- [3] P. C. Fischer, A. R. Meyer, and A. L. Rosenberg. Real-time simulation of multihead tape units. *J. ACM*, 19:590–607, 1972.
- [4] O. Goldreich, S. Micali, and A. Wigderson. How to play ANY mental game. In *STOC*, 218–229, 1987.
- [5] O. Goldreich and R. Ostrovsky. Software protection and simulation on oblivious RAMs. *J. ACM*, 43:431–473, 1996.
- [6] M. T. Goodrich and M. Mitzenmacher. Privacy-preserving access of outsourced data via oblivious RAM simulation. In *ICALP*, 576–587, 2011.
- [7] M. T. Goodrich, M. Mitzenmacher, O. Ohrimenko, and R. Tamassia. Privacy-preserving group data access via stateless oblivious RAM simulation. *CoRR*, abs/1105.4125, 2011.
- [8] A. Kirsch, M. Mitzenmacher, and U. Wieder. More robust hashing: cuckoo hashing with a stash. *SIAM J. Comput.*, 39:1543–1561, 2009.
- [9] S. R. Kosaraju and M. Pop. De-amortization of algorithms. In *COCOON*, 4–14, 1998.
- [10] E. Kushilevitz, S. Lu, and R. Ostrovsky. On the (in)security of hash-based oblivious RAM and a new balancing scheme. Cryptology ePrint Archive, Report 2011/327, 2011.
- [11] R. Pagh and F. Rodler. Cuckoo hashing. *Journal of Algorithms*, 52:122–144, 2004.
- [12] E. Stefanov, E. Shi, and D. Song. Towards Practical Oblivious RAM. *CoRR*, abs/1106.3652, 2011.
- [13] P. Williams and R. Sion. Usable PIR. In *NDSS*, 2008.
- [14] P. Williams, R. Sion, and B. Carbunar. Building castles out of mud: practical access pattern privacy and correctness on untrusted storage. In *CCS*, 139–148, 2008.