Capturing Lombardi Flow in Orthogonal Drawings by Minimizing the Number of Segments

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1 Introduction

An aesthetic property prevalent in Lombardi's art work is that he tends to place many vertices on consecutive stretches of linear or circular segments that go across the whole drawings. This creates a metaphor of a "visual flow" across a drawing. Inspired by this property, we study the following problems for orthogonal drawings of planar graphs (see Fig. 1)[1]:

1. A *minimum segment orthogonal drawing*, or *MSO-drawing*, of a planar graph G is an orthogonal drawing of G with the minimum number of segments.
2. A *minimum segment cover orthogonal drawing*, or *MSCO-drawing*, of G is one with the smallest set of segments covering all vertices of G.

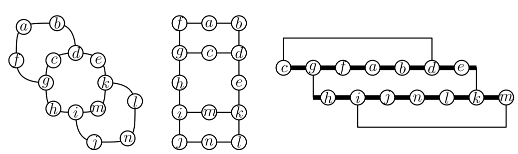

Fig. 1: (a) A planar graph G, (b) an MSO-drawing, (c) an MSCO-drawing of G.

There is a lot of prior work on minimizing the number of segments in straight-line drawings and on minimizing the number of circular arcs in planar drawings. However to the best of our knowledge, this problem has never been studied before in the context of orthogonal drawings.

A recent empirical study [4] concluded that orthogonal layouts generated by traditional algorithms focusing primarily on bend minimization lack aesthetic qualities compared to manual drawings. The study suggests that, like in the

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works by Lombardi, humans prefer drawings with linear "flow" that connect chains of adjacent vertices. Our specific interest here is to study such "Lombardi flow" for orthogonal graph drawings.

We present the following results.

- We give a polynomial-time algorithm for the MSO-drawing of an embedded plane graph, using the network-flow algorithms [2,3,5] for minimizing bends.
- We show that finding MSCO-drawing is NP-hard even for degree-3 graphs.
- For trees and series-parallel graphs with maximum degree 3, we provide polynomial-time algorithms for upward orthogonal drawings with the minimum number of segments covering the vertices.

2 Detailed List of Main Results

In this section, we give a detailed list of our main results; please see the full version of the paper [1] for proofs. Recall that the *2-factor* of a graph G is obtained by repeatively identifying each degree-2 vertex to one of its neighbors.

Lemma 1. *Let G be a plane graph with maximum degree 4 and let G' be the 2-factor of G. Then G' has an orthogonal drawing with b bends if and only if G has an orthogonal drawing with $b + k/2$ segments, where k is the number of odd-degree vertices in G.*

We find an MSO-drawing of G by computing a minimum-bend drawing of the 2-factor, using the $O(n^{1.5})$-time algorithm by Cornelsen and Karrenbauer [2].

Theorem 1. *For an embedded n-vertex planar graph G, with maximum degree 4, an MSO-drawing of G can be computed in $O(n^{1.5})$ time.*

Let Γ be an orthogonal drawing for a planar graph G with maximum degree 4. A set of segments S in Γ is said to *cover* G, if each vertex in G is on some segments in S. The *segment-cover number* of Γ is the minimum cardinality of a set of segments covering G. Given a planar graph G, with maximum degree 4, a *minimum segment cover orthogonal drawing* or *MSCO*-drawing of G is an orthogonal drawing with the minimum segment cover number.

Theorem 2. *Finding a minimum segment cover orthogonal drawing for a planar graph G is NP-hard, even if G is a planar graph with maximum degree 3.*

Theorem 3. *Let G be a series-parallel graph with maximum degree 3 with the SPQ-tree $\mathcal{T}$ and let $\#(P^*)$ be the number of P-nodes in $\mathcal{T}$ with at least two S-nodes as children. If the root of $\mathcal{T}$ is a P-node with three S-nodes as children, then $\#(P^*) + 2$ segments are necessary and sufficient to cover all vertices of G in any upward orthogonal drawing of G; otherwise $\#(P^*) + 1$ segments are necessary and sufficient.*

Algorithm for Rooted Trees. Let T be a rooted tree. Take two copies T_1, T_2 of T. Identify the two copies of each leaf, to obtain a series-parallel graph with the maximum degree 3. An upward orthogonal drawing of this graph by the above algorithm also gives an upward drawing of T with optimal segment-cover.

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