Inverse-Rendering-Based Analysis of the Fine Illumination Effects in *Salvator Mundi*

Marco (Zhanhang) Liang, Shuang Zhao and Michael T. Goodrich

Marco (Zhanhang) Liang
Student
Department of Computer Science
Donald Bren School of Information
    and Computer Sciences
University of California, Irvine
3019 Donald Bren Hall
Irvine, CA 92697, U.S.A.
zhanhanl@uci.edu

Shuang Zhao
Professor
Department of Computer Science
Donald Bren School of Information
    and Computer Sciences
University of California, Irvine
3019 Donald Bren Hall
Irvine, CA 92697, U.S.A.
shz@ics.uci.edu

Michael T. Goodrich
Professor, Corresponding Author
Department of Computer Science
Donald Bren School of Information
    and Computer Sciences
University of California, Irvine
3019 Donald Bren Hall
Irvine, CA 92697, U.S.A.
goodrich@uci.edu

ABSTRACT

The painting *Salvator Mundi* is attributed to Leonardo da Vinci and depicts Jesus holding a transparent orb. The authors study the optical accuracy of the fine illumination effects in this painting using inverse rendering. Their experimental results provide plausible explanations for the strange glow inside the orb, the anomalies on the orb and the mysterious three white spots, supporting the optical accuracy of the orb's rendering down to its fine-grain details.

Salvator Mundi is a painting attributed to Leonardo da Vinci, dated to circa 1500. (See Fig. 1a.) Although this painting is over 500 years old, it was restored and given expert authentication only recently. It was auctioned in 2017 for $450.3 million, becoming the most expensive painting ever sold [1,2].

Salvator Mundi depicts Jesus in Renaissance robes holding a transparent orb. This orb may appear to be a simple crystal ball, but there is debate in the art history and scientific literature about its composition and optical accuracy, especially since the painting dates to when Leonardo was studying optics [3]. Several observers have noted that the orb is not rendered as a solid crystal orb, for instance, which would invert and magnify the robes behind the orb. The historian Isaacson [4] writes that Leonardo deliberately rendered the orb inaccurately, while Kemp [5], who helped authenticate the painting, writes in the science journal *Nature* that the orb "glistens with points of light" that are not "spherical bubbles found in glass." Noest's reply [6] in *Nature* questions this on scientific grounds, noting a lack of optical distortion in the orb, and he hypothesizes that the mysterious white spots were instead painted on the orb. More recently, Hope [7] argues against an attribution to Leonardo on the grounds that he considers *Salvator Mundi* to be optically inaccurate. Thus, debate regarding the optical accuracy of *Salvator Mundi* is continuing and is still influencing expert opinion regarding authentication of the painting.
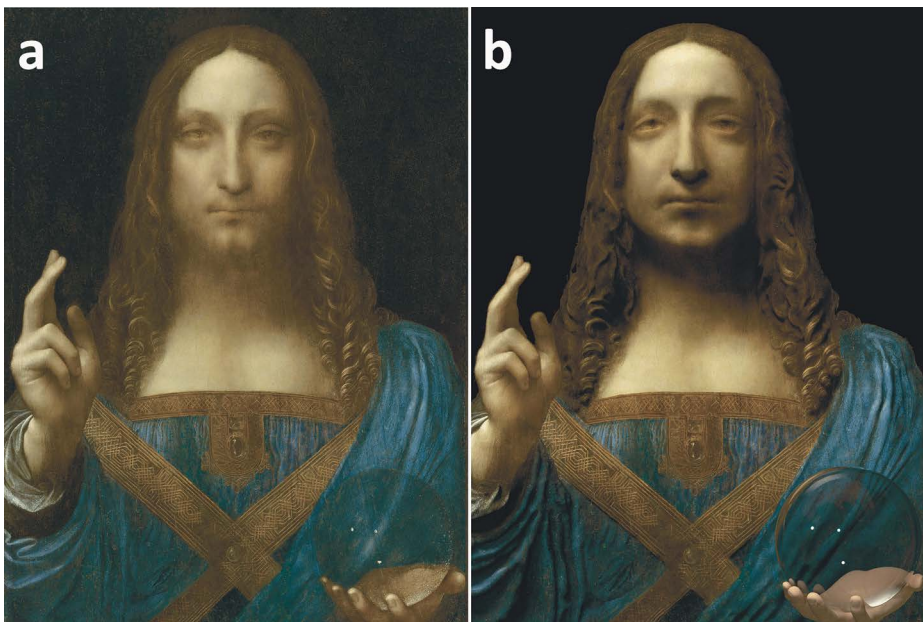

Fig. 1. (a) Leonardo da Vinci?, *Salvator Mundi*, ca. 1500. (public domain image) (b) Synthetic rendering of *Salvator Mundi* using a 3D setup and the Mitsuba physically based renderer. (Mitsuba developer: Wenzel Jakob. Left hand model © SuperDasil on BLEND SWAP. Synthetic rendering © Marco Liang.)

Although this painting dates to a time when the printing press was still a novelty and even the slide rule hadn't yet been invented, we can nevertheless use computational tools to address this debate, as has been done to address other questions in art history [8–11]. For example, *inverse-rendering* analysis uses computer graphics to infer scene information from photographs [12–15]. We felt it was appropriate to use this technique for our experiments, even though *Salvator Mundi* is not a photograph, because of Leonardo's general attention to detail and interest in optics [16] and also because of the details found in the rendering of the orb itself, which we discuss in detail below.

With respect to related work, Grundy [17] studies the angle at which *Salvator Mundi* was originally hung using image-rendering filters. In previous work [18], we used inverse-rendering analysis
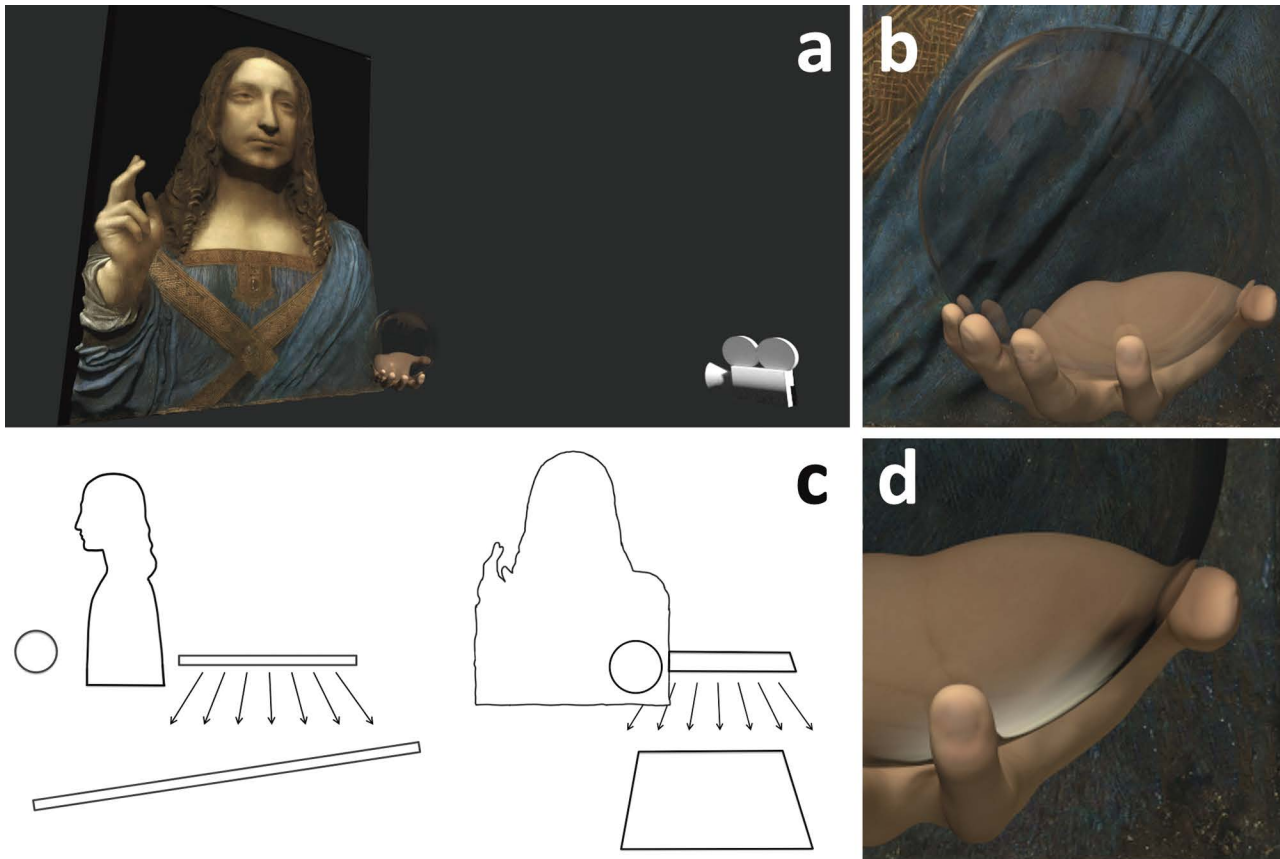
Fig. 2. (a) Virtual scene setup. (b) Rendering of a hollow orb. (c) Side view and front view of dark field lighting setup, with an area light behind the relief and a tilted Lambertian surface below to diffuse light from the side. (d) Rendering of the orb with dark field lighting. (© Marco Liang)

showing the orb in *Salvator Mundi* to be hollow, but this previous work did not address the orb's fine illumination effects, such as its strange glow, detailed anomalies or three mysterious white spots.

For the present study, we performed an inverse-rendering analysis of *Salvator Mundi* to study its fine illumination effects. We formulated a 3D setup for the scene in the painting and used the Mitsuba [19] physically based renderer (PBR) [20] to produce various renderings of his setup with different lighting sources, material properties and fine-grain geometries. By then comparing these synthetic images with the painting, we qualitatively tested various hypotheses regarding the fine illumination effects in the painting with respect to materials and light sources *Leonardo?* [21] might have used (see Fig. 1b).

Setup

The goal of our experiments was to test hypotheses regarding the strange glow above the palm, the detailed defects on the orb and the mysterious three white spots. Thus, we developed a 3D virtual scene with sculpted geometries similar to the painting and experimented with different configurations using detailed models for the orb and the hand holding the orb, accompanied by a rough proxy (i.e. a sculpted three-dimensional relief) to approximate the subject's body geometry. Using the left hand as a reference, we used a sphere that was 6.8 cm in radius as the orb and set a virtual perspective camera, representing the viewing eye, 120 cm in front of the orb, acting as a perspective viewpoint to approximate a viewing configuration in a studio. With the orb and camera fixed, we located the relief 25 cm behind the orb and scaled it according to the hand to match the relative size and position in the painting. Our virtual scene configuration is illustrated in Fig. 2a.

In addition, we modeled the orb as hollow with a thickness of 1.3 mm and a refractive index of 1.51714, based on previous results [22], and used a fine polygonal mesh to reproduce the orb's smooth surface (Fig. 2b). To generate a background-darkening effect like that in the painting, we slightly decreased the light transmission rate of the orb material. As for the orb-holding hand, we finely tuned the palm using the 3D modeling and animation system Autodesk Maya,

so that the orb gently touches the hand without overlapping. Lastly, we textured the relief with the painting and applied Gamma correction to the relief texture to better render the overall smooth appearance.

Apart from geometry, illumination is another key ingredient to visual appearance. Thus, we used key and fill light to imitate the lighting condition in the painting. Based on the observation of the brightness gradients on the subject's face, chest and hands, we placed strong key light coming from above. To soften shadow boundaries, the key light has a smooth falloff around the main direction. For the fill light, we used a dim uniform environmental light to gently illuminate the rest of the scene.

The Strange Glow

To reproduce the glow above the left hand, we investigated possible lighting setups to highlight the left hand's palm, so that the orb would have a high-contrast and sharp boundary, without brightening the fingers beneath. A dark field lighting can show the outline of a transparent object against a dark background, by placing a large light source behind from the side, so that we can have light shown at the edges of the transparent object in contrast to the color of the background. Based on this observation, we used a tilted Lambertian surface to diffuse the light emitted from an area source, creating a large light source that emits light evenly 50 cm below the orb from the side (side and front views are shown in Fig. 2c). Using this setup, we were able to reproduce the highlight on the palm without brightening the fingers beneath (Fig. 2d), showing that the highlight can be a specular reflection on the orb's surface. Thus, this setup provided a plausible explanation for the strange glow.

Defects on the Orb

With the virtual scene in place, we explored possible causes of the defects on the orb's surface by experimenting with various defect shapes. Bumps are commonly found on glass surfaces, while melting or soda can introduce bubbles. We modeled five possible defect shapes: "bump," "dip," "bubble," "bump-inside" and "dip-inside," with the first two modeling bumps and dips on the exterior surface and the latter two on the interior. Bubbles are modeled as oblate spheroids; bumps are modeled as thin cylinders with rounded edges; see Figs 3b and 3c.

To find the most probable defect shape, we augmented our setup with two representative lighting setups to test our hypotheses and filter out unlikely shapes. Since the specular highlights on the different hypothesized defects are affected by two interdependent factors—the defect's shape and the light direction—we can use uniform light to produce highlights on the defect's boundary without knowing specific lighting directions; that is, general directions are sufficient. If the testing defects show no highlights on their upward-facing sides (as can be seen in a close-up view of the painting) under uniform illumination (where light comes from all directions), they wouldn't produce any highlight under more realistic lighting conditions (Fig. 3a) (where light only comes from some directions). Then, we further tested the defects with dark field lighting, which can generate specular reflection on the orb's bottom similar to that in the painting.

We first used uniform light to generate highlights on the defects' boundary. In the rendering results, no highlight shows up on the dip or bubble's upward-facing side (Fig. 3b). Also, in Fig. 3b, bubbles on top exhibit the clothes and palm with two distinct rows of color, differing from the single color rendered on the defects' body in the painting. On the contrary, bump-inside, dip-inside and bump are able to generate highlights on their top and left edges (Fig. 3c).

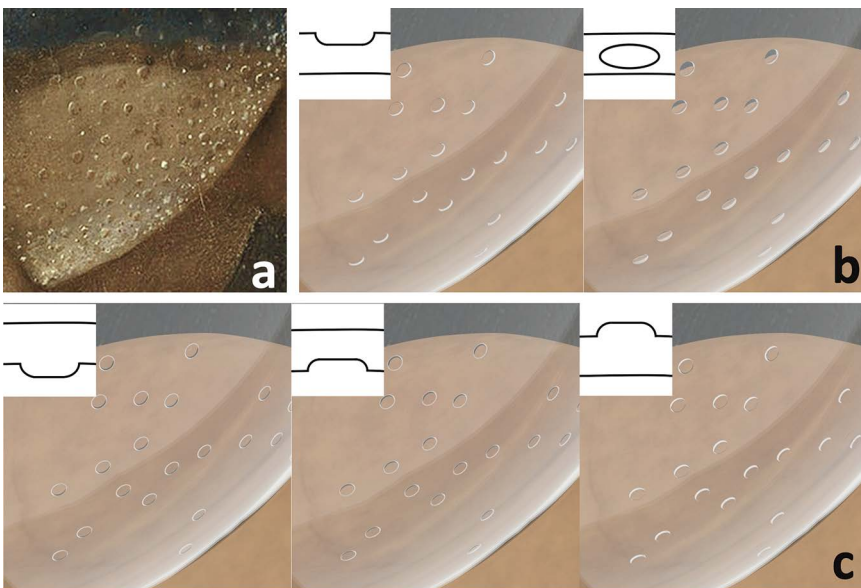With dip and bubble being unlikely, we further examined the remaining shapes with

Fig. 3. (a) Defects on the orb's surface from the painting. (b–c) Five renderings of various defects with uniform lighting. (Five renderings © Marco Liang)

dark field lighting. The results show that the bump-inside has a dark-and-thick boundary on its downward-facing side (Fig. 4a), while the dip-inside presents no highlight on its upward-facing side (Fig. 4b), demonstrating that neither the bump-inside nor the dip-inside is likely. The bump, however, renders highlight on its upward-facing side and has a thin boundary on its downward-facing side that seemingly blends into surrounding surfaces (Fig. 4c). Thus, this experiment found that "bump" was the most likely defect geometry.

We next analyzed the highlight on the bump's upward-facing side to see what casts highlights on the bump's rounding edge. We used the industry-standard approach of tracing rays inversely from the viewing "eye" toward the orb, using COMSOL Multiphysics, a general-purpose multiphysics simulation system, for this task.

Fig. 4. Three renderings of various defects with dark field lighting. (© Marco Liang)

The simulation results suggest that the highlight could be a specular reflection generated by light sources coming from above the viewer (Fig. 5a) or transmitted light of the bump coming from below (Fig. 5b). Among transmitted light, the majority (shown in orange) comes from the orb's bottom, while the other transmitted light originates outside the orb (Fig. 5b). These analyses propose that the orb-holding palm is a light source while the Lambertian surface in dark field lighting is a secondary light source for creating the highlight on the defect's upward-facing side. Using dark field lighting, we were able to reproduce the defects as bump with highlights on their upward-facing side (Fig. 5c). Based on the rendering, we inferred the defect was most likely a bump (by assuming the lateral surface is smooth).

The Mysterious White Spots

Finally, we investigated possible causes for the three white spots on the orb. Given that Leonardo was also a scientist studying optics circa 1500, we focused on exploring optical setups that could create the three white spots as specular highlights.

We first considered a simple setup that used an individual luminaire, such as a candle. Having one small light source to the left of the virtual camera yields specular reflections on the orb (see Fig. 6a). Among these specular reflections, two conspicuous highlights are located symmetrically around the orb's center: The left one is due to specular reflection on the orb's outer surface, while the right one is caused by light reflected from inside the orb. This simple lighting setup does not explain the white spots on Leonardo's orb (Fig. 8a), however, because his spots are not located symmetrically around the orb's center. We next explored lighting setups involving three individual light sources. Simply replicating the above simple lighting
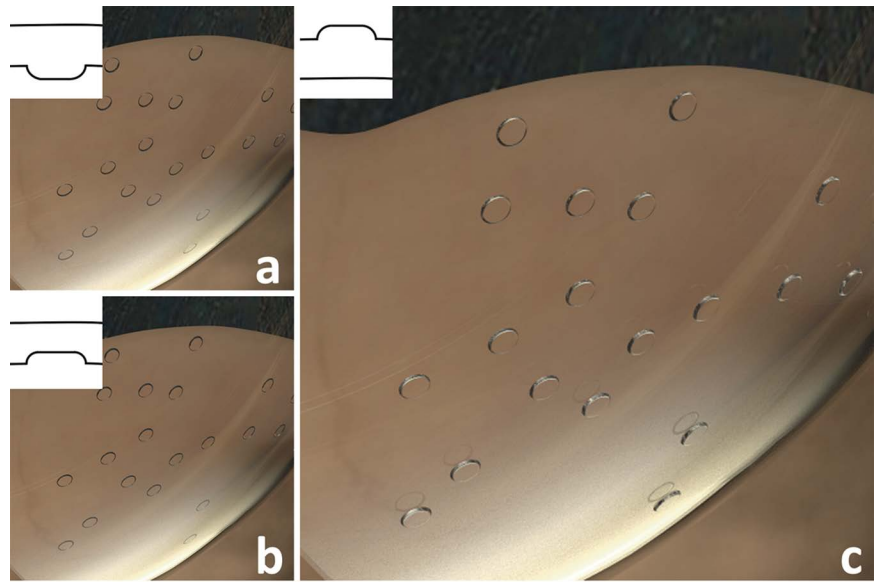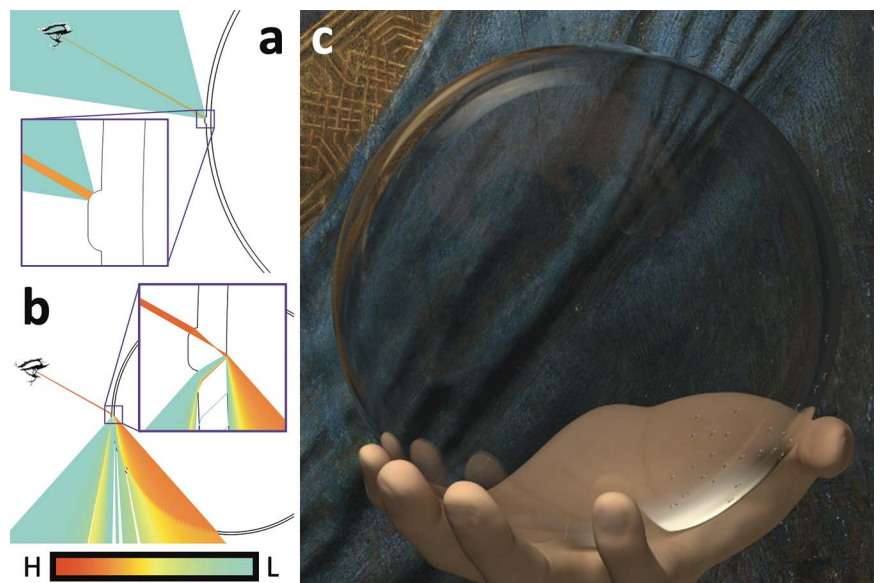

Fig. 5. (a) Analysis of specular reflection on the bump's upward-facing side. (b) Analysis of transmitted light of the bump. The orange color denotes a higher power of light (labeled "H") while the cyan color denotes a lower power of light (labeled "L"). (c) Rendering of the orb with bump defects and dark field lighting. (© Marco Liang)
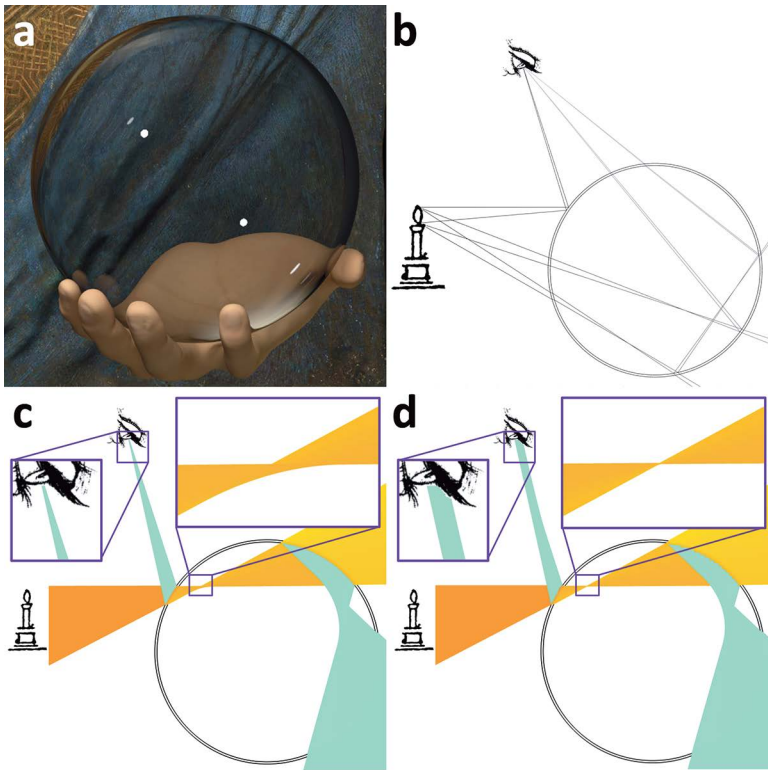
Fig. 6. (a) Rendering of a hollow orb with an unrestrained light source. (b) Specular reflections inside and outside a hollow orb with an unrestrained area light source. (c) Specular reflection on the orb, with reflected light focusing to the viewing "eye." (d) Conical incident light whose transmitted light converges to a point inside the orb. (© Marco Liang)

setup three times would not work, however, as this would create more secondary and tertiary reflections, which are not present in the orb. To find plausible configurations for light sources that could create the three bright spots, we analyzed how a glass orb interacts with light.

When light hits an orb's surface (internally or externally), a fraction is reflected while the remaining transmits through. When a light source emits light in all directions, as illustrated in Fig. 6b, multiple highlights can be seen, due to light being reflected at different locations on the orb.

To find the precise shape of incident light, we let the reflected light on the orb's surface converge to the virtual camera (the viewing "eye") by manipulating the direction of incident light. The result shows that incident light is focused conically toward the orb, and an expanding light area (in cyan inside the orb) reveals a divergent escape of transmitted light from the orb (Fig. 6c). In addition, transmitted light inside the orb converges over some region of space but does not focus to a point (Fig. 6c). If we had the transmitted light focused to a point (and thus had incident light focused to a point) (Fig. 6d), a fraction of reflected light would fall off the viewing "eye," resulting in a decrease in the highlight's area. Given these findings, we considered two setups.

Our first setup used baffles to constrain light within a conical shape, as illustrated in Fig. 7a. Specifically, we used a trapezoidal baffle, a 1-meter-long square pyramid with the top chopped off, to form a light channel. To further limit the spread of light exiting the baffle, which is necessary to fully avoid undesired additional reflections, we subdivided the baffle tunnel into a few smaller channels using 20 vertical and horizontal barriers (Fig. 7a). This trapezoidal baffle was then positioned between an area light source and the orb, set 1 m away from the orb to avoid it casting undesired shadows or entering the virtual camera's field of view (Fig. 7b). With this setup, a physically accurate simulation of light transport demonstrated it is possible to create the three white spots without extra specular reflections (Fig. 8c). Still, this setup involves the use of cumbersome baffles.

We then considered a more physically plausible setup that uses concave reflectors to concentrate light into the desired conical shape. This setup used elliptical reflectors and was inspired by Leonardo's notes of parabolic mirrors for exploiting solar energy [23]. To obtain the desired conical shape, we exploited a property of elliptical reflectors: Any light ray that originates from one focus point is reflected to pass through the other focus point (Fig. 7c). We utilized this property to design our setup by using an elliptical mirror with the first focus point outside the orb and the second slightly beneath the orb's surface. Specifically, we created a partial elliptical mirror with the two foci located 1 m away from each other; we positioned the light source at the first focus point, and we positioned the second focus point a few millimeters beneath the orb's surface. That way, the reflected light was guaranteed to be concentrated toward the orb. Because real light sources such as candle flames have finite extents that can cause reflected light to "defocus" at the second focus point, we created the point light source by enclosing a small area light (e.g. a candle) with a pinhole box (Fig. 7d). We used a 2 cm pinhole box with 3.5 mm diameter pinhole to enclose the light to effectively transform it into a point source. This pinhole was then put at the first focus point with the opening facing the elliptical mirror. Using this setup, we accurately reproduced the three white spots along with the defects on the orb (see Fig. 8c).

An Alternative Theory

In addition to validating our highlight-generating optical setups, we used physics-based simulation to test an alternative theory regarding the orb. This theory, proposed by Noest, hypothesizes that the three white spots are painted on the orb's

surface [24]. We placed three spherical caps with white diffuse reflectance on the orb's surface. Our simulation (Fig. 8b) indicates this is physically plausible (with a hollow orb). However, our earlier experiments have demonstrated that similar effects can be obtained optically; Leonardo would not have needed to paint on the orb to get the highlights.

Conclusion

We have provided an inverse-rendering analysis that provides plausible optical explanations for the fine illumination effects in *Salvator Mundi*. Indeed, one can even see a reflection of the index finger on the orb in the painting similar to a reflection in our renderings. Our hope is that attribution experts such as Hope [25]—who dismiss the painting's attribution to Leonardo based in part on thinking the orb was painted optically inaccurately—might reconsider their opinions based on our work and Leonardo's demonstrated interest in optics. Indeed, even though our analysis differs with Kemp's [26] in terms of the science, our analysis supports Kemp's main argument for authenticating the painting as Leonardo's based on its optical accuracy and Leonardo's study of optics.
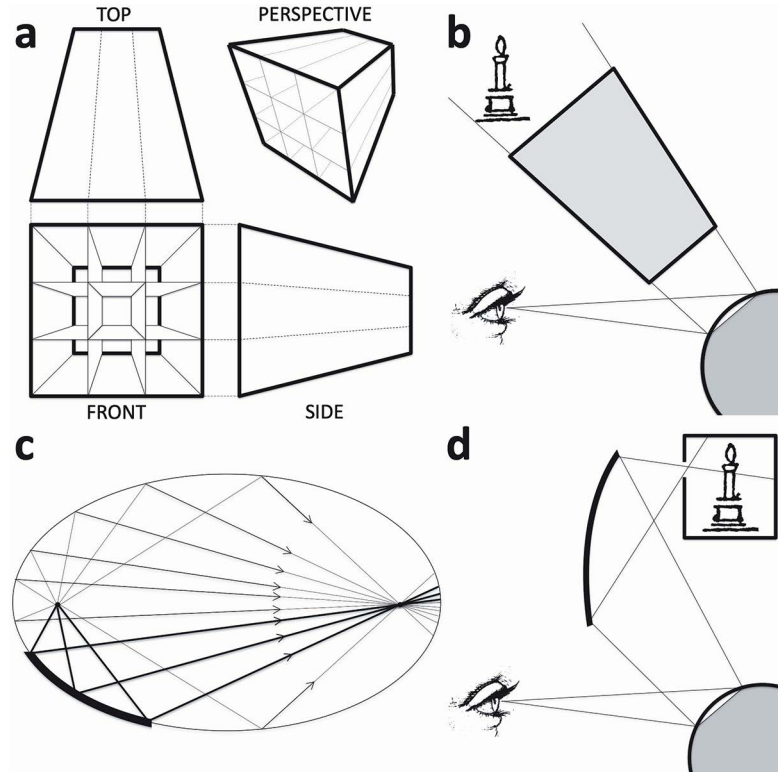

Fig. 7. (a) A simple trapezoidal baffle. (b) Position for a baffle. (c) Reflective property of an elliptical mirror. (d) Positions for an elliptical reflector and pinhole box. (© Marco Liang)
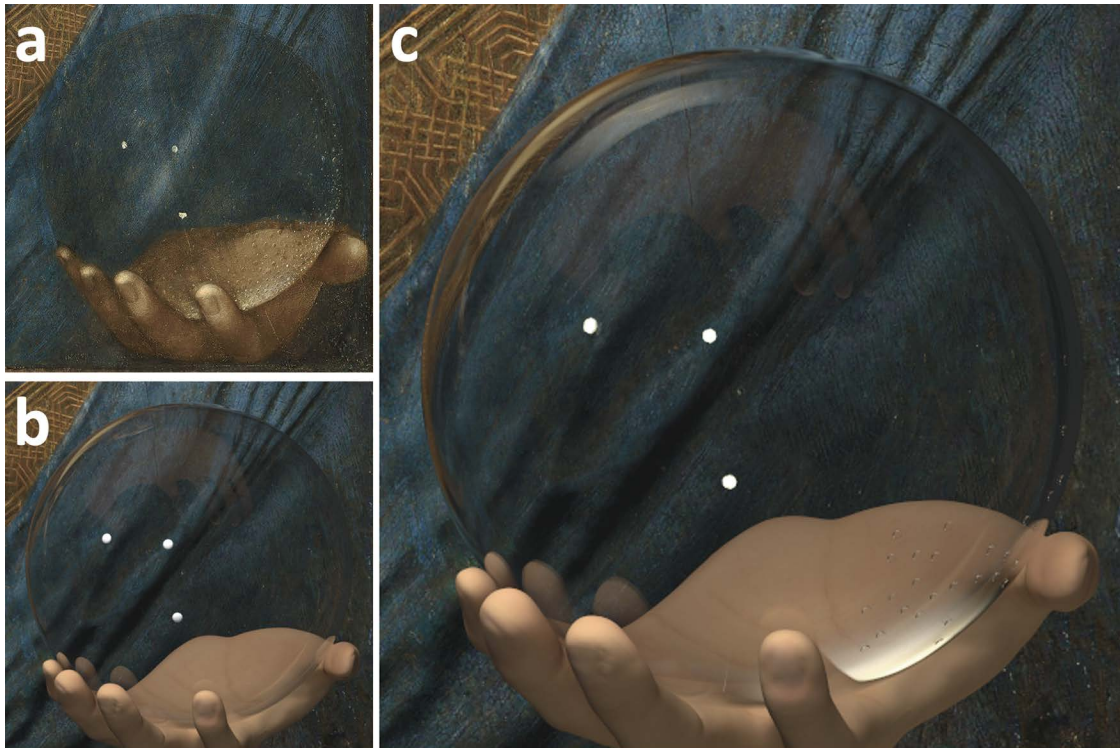

Fig. 8. (a) The orb from *Salvator Mundi* for reference. (b) Rendering of the orb with three painted specks on its surface. (c) Rendering of the orb with three white spots regenerated with concave reflectors, with bump defects as well as dark field lighting. (Two renderings © Marco Liang)

References and Notes

1. N. Freeman, "Leonardo da Vinci's 'Salvator Mundi' Sells for $450.3 M. at Christie's in New York," *ARTNEWS*, 15 November 2017.
2. B. Lewis, *The Last Leonardo: The Secret Lives of the World's Most Expensive Painting* (New York: Ballantine, 2019).
3. W. Isaacson, *Leonardo da Vinci* (New York: Simon & Schuster, 2017).
4. Isaacson [3].
5. M. Kemp, "Sight and Salvation," *Nature* **479** (2011) pp. 174–175.
6. A. Noest and M. Kemp, "Art History: No Refraction in Leonardo's Orb," *Nature* **480** (2011) p. 457.
7. C. Hope, "A Peece of Christ," *London Review of Books* **42**, No. 1 (2020).
8. A. Criminisi, M. Kemp and A. Zisserman, "Bringing Pictorial Space to Life: Computer Techniques for the Analysis of Paintings," in *Digital Art History: A Subject in Transition* (Bristol, U.K.: Intellect Books, 2005) pp. 77–99.
9. D. Hockney and C. Falco, "Optical Insights Into Renaissance Art," *Optics & Photonics News* **11**, No. 7, 52–59 (2000).
10. D. Stork, "Optics and Realism in Renaissance Art," *Scientific American* **291**, No. 6, 76–83 (2004).
11. D. Stork, "Computer Vision and Computer Graphics Analysis of Paintings and Drawings: An Introduction to the Literature," in *Computer Analysis of Images and Patterns*, Xiaoyi Jiang and Nicolai Petkov, eds. (Münster: Springer, 2009) pp. 9–24.
12. S. Boivin and A. Gagalowicz, "Inverse Rendering from a Single Image," *Conference on Colour in Graphics, Imaging, and Vision* (2002) pp. 268–277.
13. S. Marschner and D. Greenberg, "Inverse Lighting for Photography," *Color and Imaging Conference* **1** (1997) pp. 262–265.
14. R. Ramamoorthi and P. Hanrahan, "A Signal-Processing Framework for Inverse Rendering," *SIGGRAPH* (2001) pp. 117–128.
15. Y. Yu et al., "Inverse Global Illumination: Recovering Reflectance Models of Real Scenes from Photographs," *SIGGRAPH* (1999) pp. 215–224.
16. Leonardo also used a *sfomato* technique, which blurs some edges in a painting, such as is seen in the jawline of Jesus in *Salvator Mundi*. However, the *sfomato* has not been used with the orb.
17. S. Grundy, "The Distortion of Leonardo da Vinci: How Experts Missed the Significant Anamorphic Design of *Salvator Mundi*," *South African Journal of Art History* **33**, No. 4, 47–56 (2018).
18. M. Zhanhang Liang, M. Goodrich and S. Zhao, "On the Optical Accuracy of the *Salvator Mundi*," arXiv:1912.03416 (2019).
19. W. Jakob, Mitsuba 0.5, physically based rendering software tool, 2010: http://www.mitsuba-renderer.org/index_old.html (accessed April 2020).
20. M. Pharr, W. Jakob and G. Humphreys, *Physically Based Rendering: From Theory to Implementation* (Cambridge, MA: Morgan Kaufmann, 2016).
21. Because some experts disagree about whether this painting should be attributed to Leonardo, we use "*Leonardo?*" when referring to the artist who painted *Salvator Mundi*.
22. Liang, Goodrich and Zhao [18].
23. Leonardo da Vinci, Codex Atlanticus, c. 1513–15. f. 750r [277r.a].
24. Noest and Kemp [6].
25. Hope [7].
26. Criminisi, Kemp and Zisserman [8].