Animating the Polygon-Offset Distance Function*

Gill Barequet† Amy Briggs‡ Matthew Dickerson‡ Cristian Dima‡ Michael T. Goodrich†

1 Introduction

In this video, we use *The Geometer's Sketchpad^{TM}* to describe and animate a new *polygon-offset* distance function, illustrating how offset polygons grow, how they differ from scaled polygons, how distance is measured using offset polygons, why the function does not satisfy the triangle inequality, and also what a Voronoi boundary based on this distance function looks like.

Offset polygons were formally defined and used in [BBDG] in solutions to a number of geometric problems with applications to robot localization and geometric tolerancing. A similar construct was also explored by Aichholzer et al. [AAAG, AA] and given the name *straight skeletons*. In [BDG], a *polygon-offset distance function* was first formally defined based on these offset polygons (or straight skeletons) in the case when the polygon is convex. Some properties of this distance function were explored in that paper, as was the Voronoi diagram based on this distance function. Figure 1 shows a sample polygon P along with two offset polygons, one inside P and one outside P. Note that the inner offset has fewer edges than P.

The *centroid* of a polygon P is the medial axis vertex where the inner offset polygon collapses to a single point. Note that in the case of a "degenerate" polygon with two long parallel edges, the inner offset may collapse to an edge rather than a point. We thus assume a general position assumption with no degenerate polygons. Distance in this offset polygon distance function is now defined as follows: to measure distance from a point p at the centroid of polygon P, to any point q, we find the smallest offset P^* of P which contains q. Let d_u be the Euclidean distance from p to the nearest edge of P, and d^* be the Euclidean distance from p to the nearest edge of P^*. Then the offset polygon distance from p to q is given by d^*/d_u. Note that d_u is the distance the polygon must be offset inward to collapse to the centroid. Note also that the distance normalizes so that $d(p,p) = 0$, and $d(p,q) = 1$ for any point q on P.

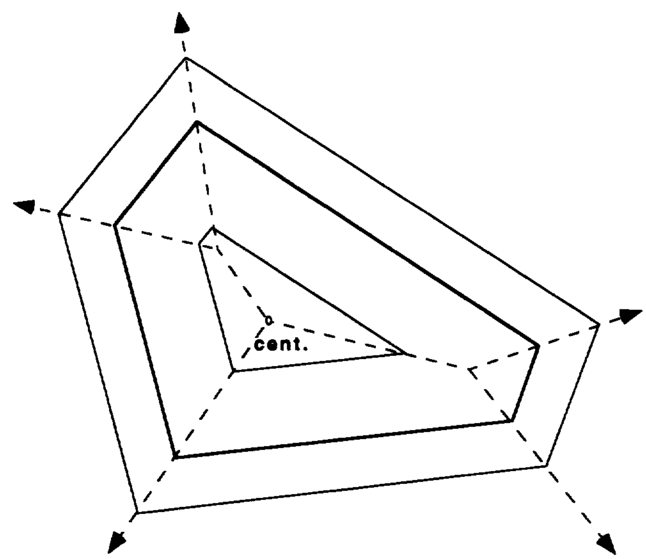

Figure 1: **Polygon with Inner and Outer Offsets.** The figure shows a convex polygon (thick lines) with its medial axis lines extended as rays to infinity(dashed lines) and an inner and outer offset polygon (thin lines).

2 Why a Polygon-Offset Distance Function

The Voronoi diagram of a set $S \subset \mathbb{R}^2$ is a well-known, powerful tool for handling a host of geometric problems dealing with distance relationships (see, e.g., [OBS]). Voronoi diagrams have been used extensively, for example, for solving nearest-neighbor, furthest-neighbor, and matching problems in many contexts. The underlying distance function used to define Voronoi diagrams is typically either the usual Eu-

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†Center for Geometric Computing, Dept. of Computer Science, Johns Hopkins University, Baltimore, MD 21218. E-mail: [barequet|goodrich]@cs.jhu.edu

‡Dept. of Mathematics and Computer Science, Middlebury College, Middlebury, VT 05753. E-mail: [briggs|dickerso|dima]@middlebury.edu

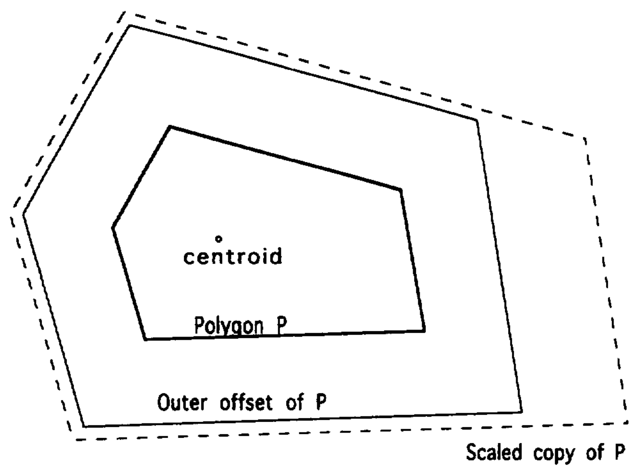

Figure 2: **Scaled and Offset Polygons.** The figure shows a polygon P (thick lines) an outer offset of P (thin lines) and a scaled version of P (dashed lines). The *centroid* of the polygon was also used as the scaling center.

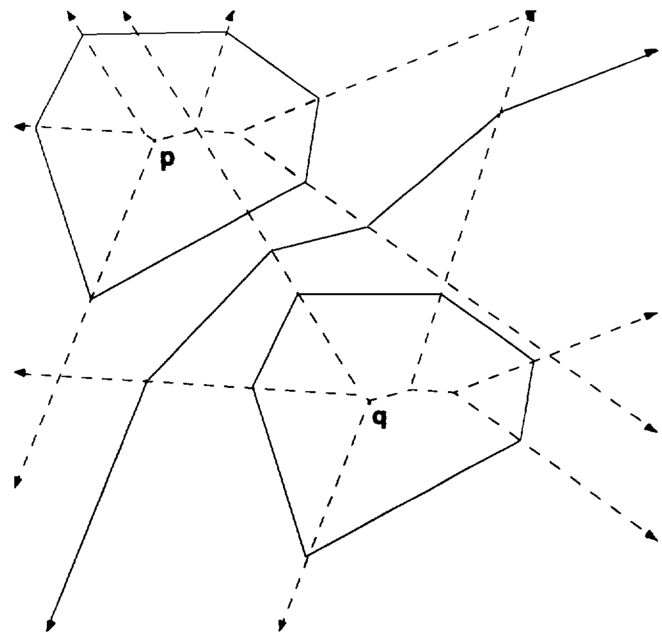

Figure 3: **Voronoi Boundary.** The figure shows two points p and q, a copy of a polygon P placed with its *centroid* on each point, and the polygon-offset distance function Voronoi polygonal bisector between the two points.

clidean metric, or more generally a distance function based upon one of the L_p metrics. There has also been some interesting work done using convex distance functions (also called Minkowski functionals [KN, p. 15]), which are extensions of the scaling notion for circles (in the Euclidean case) to convex polygons. Figure 2 shows a polygon P as well as scaled and offset polygons based on P. We feel that defining distance in terms of an offset from a polygon is actually more natural than scaling in many applications, including those dealing with manufacturing processes. This is because the relative error of the production tool (a milling head, a laser beam, etc.) is independent of the location of the produced feature relative to some artificial reference point (the origin). Therefore it is more likely to allow (and expect) local errors bounded by some tolerance, rather than scaled errors relative to some (arbitrary) center.

We thus investigate distance functions based on *offsetting* convex polygons, where the distance is measured along the infinitely extended medial-axis of such a polygon. While the scaling operation shifts each edge of the polygon *proportionally* to its distance from the origin, the offset operation shifts all the edges by the *same amount*. Offset polygons are therefore not homothetic copies of the original polygon (unless the original polygon is regular). We are interested in the investigation of basic properties of polygon-offset distance functions, with particular attention paid to how they may be used in the definition of Voronoi diagrams.

3 Summary of Video

This video illustrates this distance function, animating its behavior for several different polygons, and showing some of its properties. It is shown how the distance function differs from its relatives, the Minkowski distance functions (or scaled polygon convex distance functions). For example, the offset polygon distance function does not obey the normal triangle inequality. In fact, for colinear points, it obeys a reverse inequality.

The video ends by showing the Voronoi boundaries of two and three sites in the plane. The boundary is first shown growing along expanding offset polygons, and then shown as it would be constructed from the medial axis lines and bisectors of polygon edges.

All the animations were done using *The Geometer's Sketchpad*^{TM}, version 3.00 by *Key Curriculum Press*, which provides an excellent environment for geometric investigation.

References

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