Please answer the following questions, each of which is worth 30 points.

1. Rigorously define each of the following terms:
   (a) stable sorting
   (b) the three rank rules for red-black trees
   (c) a decision tree

2. Characterize the following recurrence relations asymptotically, using, say, the master theorem (assume each is defined so that $T(n) = 1$ for $n \leq 2$):
   (a) $T(n) = 4T(n/2) + n^2$
   (b) $T(n) = 8T(n/2) + n^2$
   (c) $T(n) = T(n/2) + n^{1/2}$

3. Briefly argue why any comparison-based sorting algorithm must run in $\Omega(n \log n)$ time.

**NOTE:** For the remainder of this test you may assume I already know about all the algorithms and data structures discussed in the text.

4. Briefly sketch a method for finding the second largest element in an $n$-element set using at most $n + \lceil \log n \rceil - 2$ comparisons. (Hint: also find the largest element.)

5. Let $A$ and $B$ be two $n$-element (unsorted) sets of real numbers. Describe an efficient method for computing the set $A \cup B$ (with no duplicates). What is the running time of your method?